Macroeconomic Disagreement in Treasury Yields *

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Abstract

I estimate a term structure model of Treasury yields where information about macroeconomic conditions is dispersed: traders form beliefs by combining prices with idiosyncratic signals about fundamentals. Econometrically, yields and inflation forecasts identify traders' information. Despite access to common public signals, beliefs are heterogeneous. Dispersed beliefs added an average of 60 basis points to ten year yields, mostly attributable to disagreement about the Federal Reserve's inflation target. Accounting dispersed information reduces the magnitude and volatility of risk premia relative to full information estimates. The estimates imply prices are moderately informative about fundamentals, and more informative about policy and others' beliefs. Keywords: Dispersed information; yield curve; macro-finance
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1 Introduction

Most macroeconomic and financial models assume agents have full information rational expectations. But professional forecasters, market participants, and managers of firms often disagree about macroeconomic conditions, and how asset prices will evolve.[1] Although ubiquitous in reality, disagreement among rational agents may be difficult to sustain in theory; as Aumann (1976) emphasizes, rational agents whose beliefs are common knowledge cannot "agree to disagree." Hence, a theoretical literature emphasizing dispersion of beliefs arising from idiosyncratic signals and a lack of common knowledge has arisen.[2]

Despite recent interest in the role of belief dispersion in macroeconomics and finance, there is a dearth of evidence on its empirical relevance for economic outcomes. A limited number of papers have structurally estimated the impact of dispersed information on business cycles.[3] While these papers have generally supported the business cycle relevance of dispersed information, others (e.g, Kasa (2000), Huo and Takayama (2014), Atolia and Chahrour (2020)) illustrate models where initially dispersed beliefs have limited practical importance.

To find more general evidence, I exploit the close link between models of business cycles and asset prices. Bond yields and the macroeconomy are closely linked both empirically and theoretically; the intertemporal Euler equation at the heart of optimizing business cycle models is a case of the canonical asset pricing condition as emphasized by Cochrane (2017). If there were no evidence of macroeconomic belief dispersion in bond yields, it would suggest economic agents should be able to use bond prices to reach consensus on macroeconomic outcomes. On the other hand, finding a non-trivial role for dispersed information in bond yields would suggest dispersion is important for asset pricing directly, as well as consumption and investment (and hence business cycles) more generally. Understanding the link between macroeconomic disagreement and asset prices, therefore, has important implications for macroeconomic models, investment finance, and monetary policy.

[1] A number of papers have documented extensive disagreement among professional forecasters and market participants using surveys. Examples include Patton and Timmerman (2010), Andrade et al. (2014) and Crump et al. (2016). Coibion et al. (2018) directly solicit the beliefs of managers of panel of New Zealand firms and documents sizeable, mistaken beliefs about macroeconomic conditions.

[2] Angeletos and Lian (2016) provide an extensive review.

[3] Melosi (2014) estimates a model where agents disagree about the exogenous money supply; Melosi (2017) estimates a version of the 3-equation Dynamic New Keynesian model where firms disagree about exogenous shocks and learn from the Federal Funds rate target. He finds it better fits the data than a full-information version of the model with markup shocks. Chahrour and Ulbricht (2019) use a combination of calibration and estimation to characterize bounds on the contributions of general deviations from full information rational expectations to business cycle fluctuations in an otherwise neoclassical model. In their benchmark, pure "sentiment" shocks explain about 57% of business cycle variation in output.

To find whether there is evidence of economically relevant macroeconomic disagreement in the yield curve, I estimate an affine term structure model (ATSM) of Treasury yields. Unlike the usual common-information assumption in the ATSM (and macroeconomics) literature, traders' information is *dispersed*: atomistic bond traders optimally combine noisy, idiosyncratic signals with public, commonly-observed bond prices to form beliefs. Because traders' expected returns depend on their beliefs about what others will pay for assets in the future, they must form expectations about others' expectations, and about others' expectations of others' expectations, and so on. This introduces the possibility of so-called Keynesian "beauty contest" motives in asset prices. Given this "forecasting the forecasts of others" (Townsend (1983)), problem, the model solution is a fixed point between the evolution of agents' beliefs and the prices that inform those beliefs. To link asset prices to the macroeconomy directly, the short rate in the model is a function of macroeconomic variables described by a structural vector autoregression (SVAR). The model is estimated with data from 1971-2007 using Bayesian methods. The precision of agents' private signals (which determines whether they influence beliefs and hence prices) is identified using the structural assumptions behind the ATSM model, optimal signal extraction, and data on both bond yields and individual inflation forecasts from the *Survey of Professional Forecasters*. Hence, the degree of information frictions, and whether those frictions result in agents disagreeing, is driven by the data.

Brief summary of the empirical contribution The estimates imply agents' beliefs are meaningfully dispersed despite the availability of endogenous asset prices that aggregate private information. An information-theoretic examination of the signal extraction problem suggests roughly half of what bond traders know about macroeconomic factors (in the model, the output gap and the difference between inflation and the Federal Reserve's inflation target) comes from observing asset prices, rather than private signals. Put another way, prices are only partially revealing about the macroeconomy. Asset prices are more informative about risks related to the central bank's unobserved inflation target, and are the source of nearly everything traders know about others' beliefs. The one period short rate, along with agents' idiosyncratic information, contains nearly all the information agents have about fundamentals and others' beliefs. The importance of the short rate in expectations formation is consistent with earlier structural and reduced form estimates of a "signaling" channel of policy over this time period (for example, Melosi (2017), Tang (2013), Nakamura and Steinsson (2018)).

The structural model allows for the decomposition of the yield curve into a component components attributable to expected average short rates (the *expectations hypothesis component)* and compensation for risk. Compensation for risk in the model takes two forms: a portion unrelated to belief heterogeneity (the *classical risk premium*) and a wedge in prices due to dispersed information (the *higher order wedge*). This wedge comes from the fact that agents' expectations of others' expectations ("higher order" expectations) differ from average expectations about fundamentals, and hence prices differ from those that would obtain if traders counterfactually held common beliefs; it can be interpreted as the contribution of Keynesian "beauty contest" motives for trading. Once we account for imperfect, dispersed information, classical risk premia are estimated to be relatively small and nearly constant, in sharp contrast to the typical results in the full-information term structure literature. The model attributes the vast majority of movement in long-term yields to the expectations hypothesis component (which is related to imperfect, but not *dispersed* information). Average short rate expectations adjust more slowly than they would if traders had full information. This is due to the fact that agents' optimal signal extraction problem attributes some changes in fundamentals to noise, and some portion of transitory shocks to persistent changes in the inflation target. The higher order wedge, on average, contributed 60 basis points to ten year yields over the sample period.

Because the risks in the model are macroeconomic, the higher-order wedge can itself be meaningfully decomposed into macroeconomic components. The majority of time variation in the wedge for long-term debt is attributable to changes in (higher-order) beliefs about monetary policy, particularly, policymakers' long-run inflation target. The decomposition suggests much of the excess sensitivity of long term yields to short-term macroeconomic news (Gurkaynak et al. (2005)) is attributed by the model to violations of the auxiliary assumption of full information. In short, the results indicate that dispersed information about the macroeconomy is a salient feature of the data when exploring the causes of business cycles, the effects of monetary policy, and bond prices in general.

Relationship to the (macro-)finance literature The dispersed information ATSM in this paper model builds that of Barillas and Nimark (2015). They assume yields are driven by latent factors in the yield curve and identify expectations using interest rate forecasts. By contrast, the model in this paper makes explicit the relationship between short rates, macroeconomic variables, financial risk, and monetary policy. This approach has the advantage of more closely relating the estimated results to the growing macroeconomic literature on dispersed information and monetary policy, which purely latent factor models for asset prices have difficulty addressing. More specifically, the model is a generalization of the structural VAR/ATSM in Ireland (2015). The central bank is assumed to select a time varying inflation target, and set short rates according to a Taylor rule. This rule responds to the output gap, deviations in inflation from the target, and financial risk. Shocks to the macroeconomic factors affecting short rates are identified using structural assumptions. Changes in market prices of risk are governed by fluctuations in a single risk variable, consistent with the results of both Cochrane and Piazzesi (2005) and Bauer (2016). Shocks to this variable are correlated with macroeconomic shocks, and the risk variable affects macroeconomic dynamics. Hence, the model allows for an interrelationship between the macroeconomy and financial markets.

This paper is particularly related to the branch of the macro-finance literature that relates long maturity bond price movements to changes in the monetary policy framework. The empirical importance of long-run inflation expectations for the dynamics of rate expectations and term premia are consistent with Gurkaynak et al. (2005), who suggest incorporating learning about a long-run inflation target can help macro-finance models explain the effect of transitory shocks on long-term bonds. The paper also complements Wright (2011) who links declines in term premia to falling inflation uncertainty, and Doh (2012), who estimates a DSGE model where agents have a noisy signal of trend inflation. Unlike those authors, I incorporate dispersed information, quantify signal informativeness, and jointly estimate the dynamics of macroeconomic variables, beliefs and bond prices.

The result that risk premia are small and short rate expectations are highly persistent contrasts with the literature that explains yields under full information rational expectations (FIRE). The results add to growing evidence that accounting for information frictions tends to make time varying risk premia less important for explaining yields. Critically, the slow adjustment of rate expectations holds even with optimal Bayesian learning where agents have model consistent beliefs and a large number of signals. This differs from other papers (e.g. Dewachter and Lyrio (2008)) where traders' forecasts are based on a model-inconsistent prior. The structural results are consistent with those of Piazzesi et al. (2013), who construct subjective beliefs without modeling inference. However, their approach does not allow for a characterization of the belief-formation process.

In addition to adding to the broad understanding of the determinants of asset prices, this paper should be of special interest to researchers working with dynamic general equilibrium or financial models featuring information frictions. The term structure model makes relatively modest structural and functional form assumptions and agents' information is consistent with the prices they observe.[4] As emphasized above, one advantage of examining macroeconomic information in the yield curve is its implications for a wide variety of equilibrium models. Barillas and Nimark (2015) show the dispersed-information ATSM nests an equilibrium model with wealth-maximizing traders. A small number of papers have embedded ATSM in DSGE models (for example Jermann (1998), Wu (2001), Doh (2012)). Like Ireland (2015), some authors have combined structural macroeconomic models with a no-arbitrage finance model under full information rational expectations. Ang et al. (2007) estimate Taylor rules in such a setting.

Outline of the remainder of the paper The next section motivates the information set-up of the model by demonstrating reduced form evidence of information frictions in financial forecasts. In section 3, I outline the asset pricing side of the model, the macroeconomic VAR. Details of the solution and estimation strategy are in sections 3 and 4. I then discuss the parameter estimates and impulse responses (section 5), the model's interpretation of the sources of yield fluctuations (section 6), and the information content of signals (section 7), before concluding.

2 Dispersed Information and financial prices: evidence from Forecasts

A number of papers (e.g. Mankiw et al. (2004), Coibion and Gorodnichenko (2012, 2015)) have shown evidence of macroeconomic information frictions using forecast data. With the exception of Coibion and Gorodnichenko (2015), most papers have focused on inflation expectations. In this section, I briefly discuss some evidence for the presence of dispersed information about the evolution of Treasury prices in particular.

FIRE places strong restrictions on forecasts. First, because agents share a common information set and a common model of the data generating process they should have a degenerate distribution; there is no scope for disagreement among forecasters. Second, forecast errors should be unpredictable. To motivate the information setup used in the model, I examine both of these predictions using financial forecasts from the Survey of Professional Forecasters

[4]This is in line with the "market consistent information" assumption advocated by Graham and Wright (2010).

(SPF).[5] Both assumptions appear to be violated.

The SPF began to survey its panel about the average yield on 3-month Treasury bills in 1981, and for 10 year Treasury bonds in 1992. Figure 1 shows the resulting dispersion of "nowcasts" (current-quarter forecasts). Since bond prices are observable in real time, the dispersion of forecasts represents disagreement about the path of yields over approximately the six or seven weeks remaining in the quarter after the survey date. The average difference between the 20th and 80th percentiles of Treasury bill forecasts from the start of the survey to the first quarter of 2020 is about 22 basis points. For the subset of data from 1987-2007 (roughly corresponding with the Great Moderation), the average range is about 19.5 basis points. Data for 10 year bonds are similar. Moreover, simple t-tests overwhelmingly reject the null of zero difference between the median forecast and the other percentiles depicted in the graph. In short, there is evidence of modest, but significant, disagreement about the average yield on short term "safe" debt in the current quarter, much less future quarters.

To more formally test for information frictions, I apply the empirical strategy in Coibion and Gorodnichenko (2015). They regress average forecast errors for horizon h steps ahead in the future on the revision from $t-1$ to t of the forecast for that period:

$$\text{Average Forecast Error}_{t,h} = \beta(\text{Average Forecast Revision}_{t,h}^{t,t-1}) + \bar{\varepsilon}_t \tag{1}$$

where $\bar{\varepsilon}_t$ is the sum of rational expectations errors.

If forecasters have FIRE beliefs (the null), the slope coefficient on revisions should be zero. Sticky- or noisy-information models imply a positive, significant regression coefficient on average revisions.[6] For Treasury bills, we overwhelming reject the null of FIRE beliefs in favor of sticky or noisy information at each horizon. For Treasury bonds (which feature a shorter sample), we reject the null in several cases, but not all. Overall, the SPF data suggests financial forecasts are better-characterized as featuring information frictions rather than FIRE. This is contrary to the typical assumptions made in term structure models.

Although the evidence in this section is partial and suggestive, there may be two concerns about these results. First, it may be that the true beliefs of bond traders differ from those of SPF participants. Hence, in the structural estimation, the beliefs of agents are also identified using both asset prices and forecasts themselves. Second, survey participants

[5] The SPF is a quarterly survey originally conducted by the American Statistical Association and the NBER before being taken over by the Federal Reserve Bank of Philadelphia in 1990. The survey is generally sent out after after the initial release of the National Income and Product Accounts to a panel of forecasters in the financial services industry, non-financial private sector, and academia.

[6] Details and full results are in appendix A.

may have a different idea in mind of what the relevant bond price to forecast is, or may not closely pay attention to financial prices. Hence, the estimation does not make use of interest rate forecasts, but rather uses inflation forecasts which are quite accurate (Faust and Wright (2013)). This ameliorates some of the risk stacking that we are the deck in favor of dispersed information. Setting these caveats aside, the analysis in this section suggests there is dispersed information that affects forecasts of Treasury prices. The nature of that information dispersion, and whether it affects prices directly, requires further analysis. Hence, we now turn to the affine term structure model.

3 The dispersed information model and structural VAR

In this section, I outline the model of asset prices and macroeconomic dynamics used to assess the effect of macroeconomic disagreement on prices. The asset pricing intuition and derivation in the next subsection closely follows that of Barillas and Nimark (2015); details are found in appendix B. After outlining the asset pricing model, I discuss the structural VAR.

3.1 The term structure model with heterogeneous information

Intuition: the fundamental asset pricing relationship Index bond traders by $j \in [0, 1]$. Denote $E_t^j x_t = E[x_t|\Omega_t^j]$ as the expectation conditional on j's information set at time t. Call Ω_t the "full information" information set (i.e., the history of the realizations of all variables up to time t). Under full information, the basic bond pricing equation is

$$P_t^n = E_t[M_{t+1} P_{t+1}^{n-1}]$$

Standard results in asset pricing theory give that the nominal stochastic discount factor (SDF) M_{t+1} exists and is positive if the law of one price holds and in the absence of arbitrage (Cochrane (2005)). If we relax the common information assumption, and instead assume there is a continuum of traders $j \in (0, 1)$ with potentially heterogeneous information sets, the pricing relationship for each agent j is:

$$P_t^n = E_t^j \left[M_{t+1}^j P_{t+1}^{n-1} \right] \tag{2}$$

Both information sets (Ω_t^j) and SDFs (M_{t+1}^j) are j-specific. Centralized trading implies it is common knowledge that all agents face the same prices today and will face the same price tomorrow; because traders are atomistic, they take prices as given. However, allowing information sets and forecasts of future prices to differ across agents, while assuming today's price is common knowledge, implies equation (2) can hold with equality only if stochastic discount factors also differ.

To decide their willingness to pay for a bond, agents must form expectations of future prices. Because future buyers face the same problem, the decision to purchase a bond today depends on a conjecture about others' (future) beliefs - the Townsend (1983) "forecasting the forecasts of others" problem. More specifically, dispersion of information implies asset prices potentially depend on *higher order expectations* - expectations of expectations.[7] Assuming common knowledge of the pricing equation, joint lognormality of prices and stochastic discount factors, and constant conditional variances, one can show (appendix B) the log price of the bond (p_t^n is related to the log SDF (m_{t+1}^j):

$$p_t^n = \int E_t^j[m_{t+1}^j]dj + \int E_t^j \left[\left(\int E_{t+1}^k[m_{t+2}^k]dk + \int E_{t+1}^k[p_{t+2}^{n-2}]dk \right) \right] dj$$
$$+ \frac{1}{2}\text{Var}(m_{t+1}^j + p_{t+1}^{n-1}) + \frac{1}{2}\text{Var}(m_{t+2}^k + p_{t+2}^{n-2})$$

The price of a bond in period t is a function of the average expected stochastic discount factor in $t+1$ plus the average expectation of the average SDF and price at $t+2$, plus variances. Repeatedly recursive substitution allows us to write prices today as a function of average higher order expectations about future SDFs and variance terms. The affine term structure model outlined below is consistent with the the above assumptions. It puts additional structure on the stochastic discount factor. Doing so makes it easier to characterize how agents form higher order expectations and how those expectations affect bond prices.[8]

Short rates and higher order expectations Call x_t be a vector of exogenous factors, which we call "fundamentals". Conjecture that the one-period risk free rate r_t is

$$r_t = \delta_0 + \delta_x' x_t \tag{3}$$

Assume there are d elements in x_t. Fundamentals follow a VAR(1):

[7] The role of higher order beliefs in asset pricing is discussed by Allen et al. (2006), Bacchetta and Van Wincoop (2008), and Makarov and Rytchkov (2012).

[8] The noisy rational expectations literature in asset pricing – following Grossman and Stiglitz (1980); Admati (1985) and others – typically models the payoffs of agents, and market clearing, more explicitly. Barillas and Nimark (2015) show that the ATSM nests a model of this type. The advantage of the ATSM with dispersed information is that it allows for a large set of assets while remaining flexible and tractable enough for estimation.

$$x_{t+1} = \mu^P + F^P x_t + C\varepsilon_{t+1} \tag{4}$$

where $\varepsilon_{t+1} \sim N(0, I)$.

Each period, agents observe private signals which are a linear combination of x_t and an idiosyncratic noise component:

$$x_t^j = Sx_t + Q\eta_t^j \tag{5}$$

where $\eta_t^j \sim N(0, I)$ is assumed to be independent across agents. For tractability, and in keeping with most of the dispersed information literature, I assume signal precision is the same across all agents, fixed at all times, and common knowledge.

By the no-arbitrage condition (equation (2)), bond prices are related to stochastic discount factors, which themselves are assumed to be a function of fundamentals (x_t). Future stochastic discount factors will be a function of (future) fundamentals. Combined with the fact that bond prices *today* are functions of higher order expectations about stochastic discount factors, the relevant state vector will be the *hierarchy of average higher order expectations* about fundamentals (Nimark (2007)).[9] Denote average beliefs about a fundamental as $x_t^{(1)}$, average beliefs about the average belief as $x_t^{(2)}$, etc, so that pth order average expectations are recursively defined as

$$x_t^{(p)} \equiv \int E\left[x_t^{(p-1)} | \Omega_t^j\right] dj$$

and the hierarchy of average order expectations up to some order $\bar{k}$ is the vector X_t:

$$X_t \equiv \begin{bmatrix} x_t & x_t^{(1)} & \cdots & x_t^{(p)} & \cdots & x_t^{(\bar{k})} \end{bmatrix}'$$

Conjecture the log bond price

$$p_t^n = A_n + B_n' X_t + \nu_t^n \tag{6}$$

[9]Because of the endogenous price signals and the fact that there are more shocks than signals, there is not a closed-form solution for the dispersed information model in general (Huo and Takayama (2014)). In this paper, I use the solution method proposed in Nimark (2007), which solves the model approximately by truncating the state space in terms of orders of beliefs. Huo and Takayama (2014) propose solving this type of model in the frequency domain but also show that in general their method requires approximation when there are endogenous signals. Huo and Pedroni (2017) suggest time-domain methods are faster when the solution method involves numerically finding a fixed point. They find an exact solution for beauty context models with dispersed information and endogenous signals, but their technique is not applicable outside of that context. In the estimated results, $\bar{k} = 15$. The majority of the weight in bond prices is on the first few orders of expectation; raising the order increases the computational burden substantially without improving the fit of the model.

where ν_t^n is a maturity-specific shock, *i.i.d.* across time and maturities.[10] Further conjecture that X_t follows a VAR(1)

$$X_{t+1} = \mu_X + \mathcal{F} X_t + \mathcal{C} u_{t+1} \tag{7}$$

where u_{t+1} contains all aggregate shocks - the shocks to fundamentals ϵ_t and the vector of price shocks ν_t.

(Log) yields at time t of a zero coupon bond maturing in n periods are defined as $-\frac{1}{n}p_t^n$.[11] Collect yields in a vector $y_t \equiv \begin{bmatrix} -\frac{1}{2}p_t^2 & \cdots & -\frac{1}{n}p_t^n & \cdots & -\frac{1}{\bar{n}}p_t^{\bar{n}} \end{bmatrix}'$.

Assume agents' information sets Ω_t^j include the histories of their private signals x_t^j, the short rate r_t and a vector of bond yields out to maturity $\bar{n}$:

$$\Omega_t^j = \left\{ x_t^j, r_t, y_t, \Omega_{t-1}^j \right\} \tag{8}$$

Note that this implies that agents do not "forget" past information when making their forecasts.

Under the conjected affine form for bond prices and exogenous information, agents' signals will also be affine in the state X_t. The filtering problem of an atomistic agent j has the following state-space representation:

$$\begin{aligned} X_{t+1} = & \quad \mu_X + \mathcal{F} X_t + \mathcal{C} u_{t+1} \\ \underbrace{\begin{bmatrix} x_t^j \\ r_t \\ y_t \end{bmatrix}} = & \quad \mu_z + D X_t + R \begin{bmatrix} u_t \\ \eta_t^j \end{bmatrix} \end{aligned} \tag{9}$$

If we assume that agents are Bayesian learners who have observed an infinitely long history of signals, their beliefs are described by a steady-state Kalman filter (Harvey (1989)).[12] The details of the bond trader's Kalman filtering problem are in appendix B.2. The matrices $\mathcal{F}, \mathcal{C}$ in the agents' state-space problem describe how how higher order expectations evolve. These matrices depend on the individual filtering problem of traders, and equilibrium ex-

[10] The role of maturity-specific shocks is to allow for prices to not be fully revealing of the state. As $\sigma_\nu \to 0$, prices become an invertible function of the state and are hence revealing of aggregate information.

[11] Hilscher et al. (2014) document that the vast majority of Treasury debt currently held by the public has maturity of less than ten years. In the application, I set $\bar{n} = 40$, i.e., 10 years is the maximum traded by agents or used to form forecasts.

[12] Assuming Bayesian learning is consistent with the "Harsanyi doctrine" (Harsanyi (1968). This is in contrast to assuming agents have model-inconsistent beliefs, as in the "agree-to-disagree" literature (Harrison and Kreps (1978)). Assuming an infinite signal history is standard in the dispersed information literature, and obviates the need to track an infinite number of atomistic individuals' information sets.

pressions for prices (detailed below). Prices themselves depend on the evolution of (higher order) expectations. Aggregating across traders implies a fixed point expression for $\mathcal{F}$ and $\mathcal{C}$ (appendix equation (30)).

SDFs and bond prices Deriving an expression for prices requires explicitly modeling bond traders' SDFs. As is common in affine term structure models, I assume stochastic discount factors are essentially affine (Duffee (2002)). The log SDF is assumed to take the form:

$$m_{t+1}^j = -r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - \Lambda_t^{j\prime}a_{t+1}^j \tag{10}$$

In the above expression, $\Lambda_t^{j\prime}$ are (time-varying) market prices of risk for holding bonds. a_{t+1}^j is the vector of one-period-ahead bond price forecast errors, which have unconditional covariance matrix Σ_a.

$$a_{t+1}^j \equiv \begin{bmatrix} p_{t+1}^1 - E_t^j[p_{t+1}^1] \\ \vdots \\ p_{t+1}^{\bar{n}-1} - E_t^j[p_{t+1}^{\bar{n}-1}] \end{bmatrix} \tag{11}$$

These errors occur because of shocks that were unanticipated by agents. Hence, the vector of forecast errors span the risks that agents must be compensated for.

Assume prices of risk Λ_t^j are an affine function of X_t^j and the vector of maturity shocks:

$$\Lambda_t^j = \Lambda_0 + \Lambda_x X_t^j + \Lambda_\nu E[\nu_t|\Omega_t^j] \tag{12}$$

where X_t^j is are trader j's expectations (from 0 to $\bar{k}$) of the latent factors

$$X_t^j \equiv \begin{bmatrix} x_t^j \\ E_t^j[x_t|\Omega_t^j] \\ \vdots \\ E_t^j[x_t^{(\bar{k})}|\Omega_t^j] \end{bmatrix} \tag{13}$$

As mentioned above, the prices of risk represent additional compensation required for traders to be willing to hold an additional unit of each type of risk. If Λ_x and Λ_v were zero matrices, risk premia would be constant. If $\Lambda_t^j = \mathbf{0}$, agents would be risk-neutral.

Given the conjectured bond price equation (6), appendix B derives the following recursive representation of bond prices:

$$A_{n+1} = -\delta_0 + A_n + B_n\mu_X + \frac{1}{2}e_n'\Sigma_a e_n - e_n'\Sigma_a\Lambda_0 \quad (14)$$
$$B_{n+1}' = -\delta_X + B_n'\mathcal{F}H - e_n'\Sigma_a\widehat{\Lambda}_x \quad (15)$$
$$A_1 = -\delta_0$$
$$B_1 = -\delta_X'$$

The price of a one-period bond is $p_t^1 = -\delta_0 + [\delta_x, \mathbf{0}]X_t = -r_t$. H is a matrix that selects only higher order expectations terms.[13] e_n' is a selection vector that has 1 in the n^{th} position and zeros elsewhere. $\widehat{\Lambda}_X$ is a normalization of Λ_X.[14]

Given the state-space representation for prices, beliefs, and fundamentals, it is straightforward to calculate average interest rate expectations over the life of the bond (the expectations hypothesis component), the component driven by expectations *about expectations* (the "higher order wedge") and remaining composition for risk that is orthogonal to the higher order wedge (the "classical risk premium.") Appendix B.6 contains details of the decomposition.

3.2 The macroeconomic environment and prices of risk

This section outlines the evolution of fundamentals x_t and details the prior restrictions on the VAR parameters which allow for structural interpretations of the shocks, ensure the model is identified, and constrain the estimation to economically relevant areas of the parameter space. These assumptions are similar to those of Ireland (2015).

3.2.1 Macroeconomic dynamics

Assume short rates are managed by a central bank that sets an exogenous, time varying, long run inflation target τ_t and then picks a short rate r_t to manage an interest rate gap

[13] More specifically, H is a matrix operator that replaces nth order expectations with $n+1$-th order expectations and annihilates any orders of expectation greater than $\bar{k}$. This is equivalent to writing prices in terms of a (hypothetical) agent whose SDF is equal to the average.

[14] For comparison, under full information with no maturity-specific shocks, Equations (14) and (15) are replaced by

$$A_{n+1} = -\delta_0 + A_n + B_n\mu_X - B_n\lambda_0 C + \frac{1}{2}B_n'CC'B_n \quad (16)$$
$$B_{n+1} = -\delta_x + B_nF^P - B_nC\lambda_x \quad (17)$$

$g_t^r = r_t - \tau_t$. Define the deviation of inflation from its long run target $g_t^\pi = \pi_t - \tau_t$. The evolution of the interest rate "gap" takes the form of a Taylor-type reaction function:

$$g_t^r - g^r = \phi_r(g_{t-1}^r - g^r) + (1 - \phi_r)\left(\phi_\pi g_t^\pi + \phi_y(g_t^y - g^y) + \phi_v v_t\right) + \sigma_r \varepsilon_{rt} \tag{18}$$

In this expression, g_t^y is the output gap. The latent financial risk factor v_t shifts prices of risk Λ_t^j in a manner specified below. Including v_t in the Taylor rule is a simple way to incorporate contemporaneous feedback between financial conditions and the central banks' policy stance. I impose prior restrictions on these parameters. First, I assume that ϕ_v is non-negative.[15] Second, I assume ϕ_r falls between zero and 1. Finally, I assume ϕ_π and ϕ_y are both positive.

The long-run inflation target is assumed to follow an $AR(1)$ process:

$$\tau_t = (1 - \rho_{\tau\tau})\tau + \rho_{\tau\tau}\tau_{t-1} + \sigma_\tau \varepsilon_{\tau t} \tag{19}$$

with $\rho_{\tau\tau} \in (0, 1)$.[16]

Collecting the factors in x_t:

$$x_t = \begin{bmatrix} g_t^r & g_t^\pi & g_t^y & \tau_t & v_t \end{bmatrix}' \tag{20}$$

they can be written in matrix form:

$$P_0 x_t = \mu_x + P_1 x_{t-1} + \Sigma_0 \varepsilon_t \tag{21}$$

Exact expressions for $P_0, \mu_x, P_1, \Sigma_0$ are shown in appendix B.4. Left multiplying by P_0^{-1} yields (4). After a normalization of one covariance matrix parameter, the VAR is exactly identified. I calibrate $\sigma_v = 0.01$.

Restrictions on prices of risk. The matrices governing the mapping of factors into prices of risk shown in (10) and (12) are high-dimensional. As Bauer (2016) notes, absent restrictions on the prices of risk, the estimation does not take into account cross-sectional information in the yield curve. Accordingly, I incorporate two sets of restrictions. First, I follow Ireland (2015) in imposing that, under *full* information, changes in prices of risk are

[15] While in principle unnecessary for identification, this restriction is consistent with the idea that the central bank has raised rates in response to an increase in risk premia. McCallum (2005) suggests a Taylor rule with smoothing and a reaction to the term spread - itself affected by a possibly time varying term premium - is consistent with a negative slope coefficient in Campbell and Shiller (1991) regressions.

[16] Stationarity is assumed for two, related, technical reasons. The first is that interest rate processes that contain a unit root will leave long-run yields undefined. The second, related issue, is the stationarity of asset prices helps ensure that approximation error caused by truncating $\bar{k}$ can be made arbitrarily small (Nimark (2007)).

driven by entirely by changes in v_t, and that v_t is not itself a source of priced risk.[17] Second, I follow Barillas and Nimark (2015) in restricting Λ_t^j to nest the full information version of the model without maturity shocks. This means that the same number of parameters govern prices of risk in the full and dispersed information models.[18] Details of how these restrictions are implemented are shown in appendix B.5.

3.3 Signals

The last step is to specify agents' agents' idiosyncratic signal structure. I do not formally model the information choice of traders but impose an exogenous information structure.[19] I assume prices are observed without error, but individuals' observations of the non-price factors driving prices of risk are subject to idiosyncratic noise that is uncorrelated across variables. Recalling (5), I assume bond traders observe the short rate and separate signals about inflation and the long-run inflation target. To summarize:

$$x_t^j = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \widetilde{\sigma}_\pi & 0 & 0 & 0 \\ 0 & \widetilde{\sigma}_y & 0 & 0 \\ 0 & 0 & \widetilde{\sigma}_\tau & 0 \\ 0 & 0 & 0 & \widetilde{\sigma}_v \end{bmatrix} \begin{bmatrix} \widetilde{e}_t^{\,\pi} \\ \widetilde{e}_t^{\,y} \\ \widetilde{e}_t^{\,\tau} \\ \widetilde{e}_t^{\,v} \end{bmatrix} \tag{22}$$

[17]The restrictions on v_t are consistent with the empirical results in Cochrane and Piazzesi (2008), Dewachter et al. (2014), and Bauer (2016) who all find that a single factor is responsible for nearly all time variation in bond risk premia. Cochrane and Piazzesi (2008) show that a single "tent shaped" factor extracted from the yield curve explains nearly all time variation in term premia. Dewachter et al. (2014)'s risk factor is identified by a similar assumption to that of Ireland (2015) and is highly correlated with the Cochrane-Piazzesi factor. Bauer (2016) use Bayesian methods to estimate a Gaussian term structure model and finds evidence for strong zero restrictions which imply only changes in the "slope" factor affect term premia.

[18]Like Barillas and Nimark (2015) I also assume the maturity specific shocks shocks have the same standard deviation across yields, although the shocks to each yield are independent.

[19]Exogenous information keeps the model tractable enough to allow for likelihood based estimation. The downside is vulnerability to a Lucas-critique-like argument that the allocation of attention is not invariant to policy changes, and the model does not let the precision of signals vary over the business cycle, as it might in a model where agents optimally (re)allocate attention. The advantage is this allows estimation of the precision of traders' information that is consistent with asset price movements over the sample.

4 Solution and Estimation

4.1 Solution

The solution to the model is a fixed point of the bond pricing terms (A_n, B_n) and agents' beliefs. In particular, we need to find a fixed point between the price recursions, equations (14) and (15), the mean-square error matrix for state forecast error (equation (29) in the appendix), and the law of motion for the hierarchy of average higher-order expectations (equation (30) in the appendix).

4.2 Econometric Model and Data

The model period is a quarter and the estimation runs from Q4:1971-Q4:2007. The end date is chosen prior to the zero lower bound period because the linear model does not respect the ZLB constraint.[20] I take data on (non-annualized) zero coupon yields from the yield curve estimates in Gurkaynak et al. (2007), averaged over the quarter. In the econometric model, I use the the short rate (assumed to be the Federal Funds Rate, as in Piazzesi et al. (2013)), and rates on 1,2,3,4, 5 year and 10 year bonds.[21]

To identify agents' beliefs and the macroeconomic dynamics, I use data on the output gap (calculated as the log difference between real GDP and its HP filtered trend using a smoothing parameter of 16,000)), inflation and inflation forecasts (as measured by log changes in the GDP deflator), and treat τ_t and v_t as latent process.

The estimation includes the entire cross-section of one and four quarter ahead forecasts for GDP deflator inflation from the SPF. The advantage of inflation forecasts is that they are available for the entire sample period with a relatively high response rate. Moreover, inflation forecasts in the SPF are quite accurate on average, which means this choice of data does not automatically favor sizable information frictions. Individual survey responses are treated as a noisy indicator of the *average* expectation, where the extent of the noise is pinned down by the model-implied cross-section of expectation around the first-order average expectation.[22]

[20] The recent literature on shadow rates - for example Wu and Xia (2014) and Bauer and Rudebusch (2016) - emphasizes that dynamic term structure models perform poorly when the zero lower bound is not taken into account. Moreover, the zero lower bound introduces a nonlinearity in the signal structure which the conventional Kalman filter does not take into account. Estimating shadow rate models involves either discretization of the state space or simulation-based methods which are currently computationally infeasible in this dispersed information setting.

[21] Note agents are assumed to observe the whole yield curve, not just this subset.

[22] For the full information version of the model, I treat forecasts as if they are observations of the rational

This matrix can be calculated using the Kalman filtering problem of individual agents (see appendix B.2). Because the number of respondents to the SPF has varied over time, the number of observables at different times is time varying. Accordingly, the Kalman filter equations used to estimate the likelihood of the model are time varying. Assuming there are m_t^1 respondents to the 1-period ahead question and m_t^4 to the four-period ahead question in the SPF at time t, the state space system for estimation is

$$\begin{aligned} X_t &= \mu_X + \mathcal{F}X_{t-1} + [\mathcal{C}, \mathbf{0}_{d(\bar{k}+1)\times m_t^1+m_t^4}]\bar{u}_t \\ \bar{u}_t &\sim N(0,1) \text{ with dimension } (d + (\bar{n}-1) + m_t^1 + m_t^4) \times 1 \\ \bar{z}_t &= \mu_{\bar{z},t} + \bar{D}_t X_t + \bar{R}_t \bar{u}_t \end{aligned} \tag{23}$$

where in particular $\mu_{\bar{z},t}$, $\bar{D}_t$ and $\bar{R}_t$ vary in size to account for missing observations.

I use a Bayesian Metropolis-Hastings Markov Chain Monte Carlo procedure to estimate the model parameters. Because the model has a large number of parameters and is computationally burdensome to solve, I use somewhat informative priors on macroeconomic parameters. In addition to those noted above, I impose that ρ_{yv} is non-positive, which implies that all else equal, greater risk premia are contractionary. This is consistent with most general equilibrium models with financial frictions. For similar reasons, I impose a slightly informative prior that for ρ_{yr} that is centered around -1, while still allowing the estimation to explore regions of the parameter space where this restriction does not hold. Finally, I follow Ireland in calibrating $\rho_{\tau\tau} = 0.999$. Prior distributions are reported in appendix C.2. To ensure long-run bond prices are well defined, the estimation imposes that physical and risk-neutral dynamics of bonds are stationary under full information. This means that parameter draws are only accepted if the maximum eigenvalues of F^P and $F^P - C\lambda_x$ are less than one in modulus.

I run separate MCMC chains in parallel for each model. For the full information model, each chain is of length 400,000; I discard the first 10% of each chain and subsequently analyze every 1000th draw. The DI model is much more computationally intensive; the results reported here are based on 5 chains of length $23,000$ each. I drop the first 50% of each chain (because it takes longer to stabilize) and use every 100th draw.

expectations forecast with *i.i.d.* error. I allow each forecast horizon to have a different error variance. I also treat each individual bond yield as if it were observed with maturity-specific econometric error. Conceptually, these errors are distinct from the errors in the dispersed information version. In the dispersed information model, the noise in forecasts is pinned down by the model-consistent state mean square error matrix. As discussed earlier, the maturity specific shocks are a risk faced by traders in the model, rather than being econometric noise in the empirical model.

Summary of identification The macroeconomic VAR is exactly identified by impact restrictions on the exogenous processes. Because agents are assumed to have rational expectations, their beliefs about the macroeconomic VAR are correct. Hence, any mistakes in their inference come from noise in their signals, and the extent to which they *disagree* about the state depends on the noise in their private signals. If private signals were perfectly revealing, or arbitrarily noisy, then they would receive no weight and agents would not disagree because they would condition on common information. To the extent agents disagree, it must be because their private signals are sufficiently informative to receive some weight. Hence, the cross-section of inflation forecasts is informative about private signal noise because it reveals the extent of disagreement. Agents' beliefs must also be consistent with bond prices, so yields are also informative about agents' beliefs. Given a belief process for macroeconomic variables, agents' beliefs about expected short rates are also pinned down; what remains in yields has a component that covaries with macroeconomic variables (pinning down the time-varying part of prices of risk) and an average component (the constant part of prices of risk). Prior information is also somewhat informative macroeconomic parameters.

5 Parameter Estimates and Impulse Responses

Here I report the results of the estimation for the dispersed information model. After discussing parameter estimates, I turn to impulse responses, which suggest agents have dispersed information.

5.1 Parameter estimates

Parameter estimates across chains, and posterior credible sets are reported in appendix table 9.[23] Most of the macro-VAR parameter estimates in line with the results in Ireland (2015), although estimates of the prices of risk differ. Some of this is likely attributable to differences in samples, but the full information estimates of those parameters have a high degree of dispersion, as do the parameters governing the covariance of the non-financial factors with the risk factor v. Taking the full and dispersed information parameter estimates together, it appears that it is difficult to separately identify the prices of risk terms, and the

[23]Full information parameter estimates are available upon request from the author. In terms of macroeconomic dynamics, the full and dispersed information models have relatively similar parameter estimates. This is, unsurprising, as the model does not allow for direct feedback from the inference problem of agents to the macroeconomy.

covariances that govern changes in risk.

The key parameters of interest govern the informational quality of agents (the bottom five rows of the table). The relatively small value for σ_ν implies, all else equal, that prices move mostly due to (higher order beliefs about) fundamentals rather than large direct shocks to prices. This suggests the model prefers to endogenously explain movements in yields.

Agents' idiosyncratic noise appears somewhat large in absolute terms. However, this does not necessarily imply agents' beliefs are inaccurate, because they understand the structure of the economy. For example, traders know an unanticipated increase in inflation is correlated with unanticipated increases in output ($\sigma_{y\pi} > 0$), and that higher inflation today usually depresses growth in the future ($\rho_{y\pi} < 0$). Moreover, agents learn from prices, which aggregate information. We cannot conclude simply from the parameter estimates that agents have inaccurate beliefs. All else equal, noisier private signals receive less weight.

5.2 Impulse Responses

To demonstrate some of the information mechanisms at play, I plot impulse responses for fundamentals, the first three orders of expectation about fundamentals, inflation forecasts, and prices for a subset of the model shocks. The impulse responses discussed in this section are shown for the posterior mode for clarity. The complete set of impulse responses for the dispersed information ATSM, including posterior credible sets, shown in appendix C.4.1. [24]

The fundamental impulse responses to one-standard deviation shock to the monetary policy rule are shown in figure 2. The top row displays the responses of the fundamental factors, while the subsequent rows show increasing higher-order beliefs about those variables (with inflation expectations in the far right column). For inflation and interest rates, the responses are in terms of annualized percentage points; the output gap is in percentage points, and "risk" is scaled up by 100. As expected, a shock to the short rate causes inflation and output to fall over the course of several years.

The impulse responses illustrate the identification problem faced by agents in the model. Agents observe the short rate has risen, but know this could be caused by any of the fundamental shocks. Because they are unable to discern the origin of the change, they place some posterior weight on the possibility that both inflation and the inflation target are elevated.[25] Agents persistently misattribute the cause of the increase in short rates to changes in the

[24] Full information impulse responses are omitted for space reasons, but are available on request.

[25] The rise in inflation expectations may seem puzzling. However, it is consistent with the sign-restricted VAR results of Melosi (2017) and Struby (2018); in US data, identified monetary policy shocks cause inflation expectations to rise on impact, perhaps because of signaling effects.

inflation target. Hence, they have imperfect information. However, they also believe *others* believe the inflation target has risen (second order expectations are similar to first order), and third order beliefs increase by less. This implies traders' beliefs, in addition to being imperfect, are dispersed – on average, they believe that others do not share their beliefs, especially about others' beliefs. Over time, as they observe additional information, their beliefs approach the true impulse responses (top row). In short, optimal inference in the model are characterized by mistaken beliefs about the origins of shocks and divergence of average beliefs from higher order beliefs.

Interestingly, dispersion of beliefs after a short rate shock does not have a large *direct* effect on yields (figure 3). The overall response of yields to the shock are shown in the first row. Subsequent rows show the decomposition into rate expectations, "classical" risk premia, and the higher order wedge. Average rate expectations (row 2) are elevated as a result of the shock, which explain nearly all of the increase in yields, even at the long end of the yield curve. In other words, agents may not know *why* rates have increased, but everyone agrees the path of short rates will be persistently elevated. This is driven by beliefs about the inflation target, which raises the expected path of short rates. Classical risk premia (row 3) and the higher order wedge (row 4) barely move as a result of the shock.

Because the path of expected short rates does not adjust as quickly as it would under full information, long term yields rise more after a rate shock and remain elevated. In other words, the inference problem of agents in and of itself matters for the assessment of financial fluctuations. As will be shown in section 6, once we account for the slow adjustment of expectations, yields are mostly attributed to the expectations hypothesis component.

A similar set of impulse responses for a one standard deviation increase in the inflation target τ are shown in figures 4 and 5. Movements in the inflation target cause level shifts in the yield curve by persistently raising short rates.[26] Agents are slow to adjust the shock, so the level shift is gradual, rather than immediate, but the shock to the inflation target raises yields across the board by approximately the same amount over the course of several years.

What is interesting is the difference between fundamentals (the top line in figure 4) and higher order beliefs about those fundamentals. As in the full information model, a higher inflation target is associated with a temporary expansion in output. However, agents observing higher rates, accompanied by upward movements in inflation and risk, actually believe that output rises initially, falls over the medium term, and rises again. Higher order

[26] Recall that the inflation target is the most persistent shock, with its autoregressive component calibrated to $\rho_{\tau\tau} = 0.999$.

believes follow this pattern, although third-order expectations move more dramatically.

"Risk shocks" (shocks to v) are shown in figures 6 and 7. Impulses to v_t have nearly no direct effect on output, but depress inflation, and the reduction in inflation leads output to grow over time. This result holds for both the full- and dispersed-information models, so it is not a result of the information assumption.[27]

Other impulse responses are shown in appendix C.4.1. The macroeconomic implications are as expected: shocks to the output gap induce positive comovement in inflation, output, and interest rates - in a sense they are similar to demand shocks in New Keynesian DSGE models. Inflation shocks raise output on impact but lower it over the medium term. Apart from the initial (positive) change in output, they somewhat resemble cost-push shocks to the Phillips curve in New Keynesian models.

6 Decomposing (Higher Order Expectations in) the Yield Curve

Despite the abundance of public signals, non-trivial dispersion of higher order beliefs persists in the model. What effect, if any, does this dispersion of beliefs have on prices? In this section, I use estimates of the underlying higher-order beliefs to decompose bond yields. [28] This exercise allows me to directly answer two questions: (1) What does the model attribute changes in bond yields to - changes in rate expectations, "classical" risk premia, or higher-order beliefs? (2) *Which* higher-order beliefs matter for prices? Briefly, the answer to the first question is that (slowly adjusting) rate expectations play the largest role in determining yields at all horizons. Classical risk premia are nearly constant for bonds at all maturities,

[27] The effect of risk shocks on macroeconomic variables and asset prices stand in contrast to the risk shocks in Ireland (2015). In his paper, the co-movements brought on by risk shocks are qualitatively similar to those of a monetary policy shock, albeit without a "price puzzle." Two possibilities for differences in the dynamic behavior for v_t between these results and those of Ireland (2015) present themselves. One is that this is driven by differences in sample, particularly, the difference in sample period or the presence of inflation forecasts. Removing those forecasts from the dataset does not qualitatively change the impulse responses at the posterior mode of the full information model. The second possibility is that the risk parameters and the prices of risk are not particularly well identified. Both of these explanations suggest stronger prior information might "smooth out" the posterior and make the impulse responses to risk shocks more strongly resemble those of Ireland. In the absence of strong priors, these dynamics appear to be what the data prefers.

[28] The results here are based on Kalman filtered estimates of the state, which can be thought of as inefficient estimates of the underlying hierarchy of higher-order expectations X_t. Kalman smoothing, which takes account of the whole sample to derive estimates, presents numerical problems because the one-step ahead state forecast error matrix is numerically ill-conditioned. The filtered estimates are closest to what the Kalman smoother would imply at the end of the sample.

but the importance of the higher order wedge increases in the maturity of the bond. As for the second higher order beliefs about monetary policy variables – namely, the rate gap g^r and the inflation target τ – drive most of the time variation in the wedge.

Informally, we can think about yields as be being driven by a part that is rate expectations and a residual. The path of average rate expectations (and hence, the "expectations hypothesis" part of bond yields) can be directly calculated from the state space model. Combined with our assumption of no-arbitrage, and the way risk is priced, full information rational expectations implies the part of yields unexplained by average rate expectations (the residual) is compensation for risk – the "risk premium.". However, as the decomposition outlined in appendix B.6 shows, under dispersed information the residual can be interpreted as the sum of the higher-order wedge and the gap between average expected short rates and the price that would obtain if agents counterfactually held common beliefs.[29]

Although one could focus on bonds of any maturity, here I focus on ten year yields.[30] The three-way decomposition is shown in figure 8. Comparing the top two panels, it is clear that the model attributes the majority of movement in bond yields to rate expectations. In other words, accounting for agents' subjective rate expectations makes the implied premium for investing in long term bonds less volatile. That premium is divided between the "classical" premium and the higher order wedge; they are of roughly equal magnitudes, but the former is close to constant while the wedge varies over time. The reduced importance of compensation for risk in determining bond yields is qualitatively consistent with Piazzesi et al. (2013), who use a very different methodology to arrive at this conclusion.

This result implies at least part of the dramatic failure of the expectations hypothesis is attributable to assuming agents' forecasts use full information. Accounting for agents' subjective forecasts means that volatile time varying risk premia are not as necessary to explain movements in long term yields. As for that premium, the larger, time varying portion is attributable to the the fact that agents *believe others have different beliefs*. Both classical risk premia and the higher order wedge rose during the Volcker disinflation and declined afterwards.

Why did risk premia and the higher order wedge decline over time? The model restrictions imply a decline in the classical risk premium must be attributable to a decline in v_t over time. The inflation target variable τ_t is also falling over this period and since the estimated

[29]Note also that the residual in the full and dispersed information models may be different because they imply different short rate forecasts.

[30]The results for other maturities are available upon request.

$\sigma_{v\tau} > 0$, the decline in the inflation target appears to have caused the decline in risk.[31]

We can also examine the role of higher order beliefs about these variables in determining yields. Figure 9, shows the higher-order wedge decomposed into the contributions from different higher-order beliefs about different fundamentals. The decomposition reveals that growth in the higher order wedge is attributed largely to higher order beliefs about the inflation target. A smaller contribution comes from higher order beliefs about the rate gap $g^r = r_t - \tau_t$. Since r_t is commonly observed, this means that overall, monetary monetary policy uncertainty contributes the most time variation to the wedge, at least for ten year yields. This is (partially) counterbalanced by higher order beliefs about the risk variable, which grew in the late 1970s and 80s but fell afterwards.

A plausible explanation for this change is uncertainty about the goals and credibility of the Federal Reserve prior to the Volcker disinflation. Uncertainty about the inflation target implies that people may have not only been unsure what the target was, but also what others believe the target to be, and what they believe others believe, and so on. Changes in the long-run inflation target are more important for long-run bonds because a nominal bond's real returns are eroded by sustained higher inflation. A greater commitment to fighting inflation and greater transparency lead to consensus about the Fed's current stance of policy and its implicit inflation target. This lead to the decline in the higher order wedge over time.[32]

Examining the decomposition also reveals a degree of "canceling out" of higher order beliefs. This is because different risks are not perfectly correlated with each other, and agents' higher-order beliefs are constrained by the macroeconomic environment. The average and maximum contribution of higher-order expectations to yields is shown in table 1. The contribution of higher order expectations to the wedge is increasing over the maturity of the bond. This is consistent with the model intuition at the beginning of section 3. Longer-maturity bonds are a function of future expectations of future stochastic discount factors. The longer the maturity of the bond, the larger the role of higher-order beliefs in determining

[31] Appendix figures 15a and 15b show the estimated paths of these variables both steadily declined from the early 1980s to the early 2000s.

[32] The importance of the credibility of the central bank's inflation target is consistent with Wright (2011). He argues changes in the conduct of monetary policy lowered inflation uncertainty and that inflation uncertainty significantly explains the five-to-ten year forward premium across his sample of countries from 1990-2009. Both the results here and in Wright's paper are consistent with the idea that lower inflation uncertainty over time has caused the premium on long-term US government debt to decline. In my model, this is a result of both the relationship between changes in the inflation target and the risk variable - that is, the direct role of the inflation target and risk - as well as (higher order) uncertainty about monetary policy arising endogenously from the traders' inference problem.

Table 1: Contribution of higher-order wedge to yields at the posterior mode

	1 year	2 year	3 year	4 year	5 year	10 year
Average	0.03	0.15	0.18	0.20	0.44	0.66
Maximum	0.10	0.31	0.40	0.55	0.72	0.89
	Average contributions by source:					
g_r	-0.05	-0.28	-0.41	-0.11	0.11	0.31
g_π	0.03	0.23	0.36	0.19	0.04	-0.09
g_y	0.12	0.59	0.93	0.71	0.48	0.28
τ	-0.14	-0.78	-1.18	-0.50	0.04	0.54
v	0.01	0.09	0.14	-0.08	-0.23	-0.37

the price, because additional expectations about expectations are (rationally) formed as the life of the bond increases.

Table 1 reveals how higher-order beliefs about different risks play different roles in the higher order wedge across different maturities. This is a result of the expected time path of higher order beliefs and how different risks are priced at different horizons. In particular, as figure 4 reveals, higher-order beliefs about the output gap tend to fall over the medium term when the inflation target rises, which (along with the estimated prices of risk) explains why during the period when τ contributes the most to the higher-order wedge for 10 year yields is also when g^y plays such a large role for 3 and 4 year bonds. For bonds of low maturity, the contributions of higher order beliefs are very small in absolute terms and essentially cancel out on average.[33]

[33] The contribution of higher order beliefs, and their time series properties, are somewhat different here than in Barillas and Nimark (2015). They find that higher order beliefs play a larger role in general (with the peak contribution as a fraction of yields in the early 1990s) and also find a large negative role for the higher order wedge during the early 2000s. additional restrictions the structural VAR places on the risks faced by agents. Agents' beliefs about pricing factors and the role of those factors in prices are constrained by the covariances between asset prices and macroeconomic yields in the data. Their latent factor model is more flexible. A second important difference is the choice of data. Barillas and Nimark directly use SPF data on interest rate expectations to discipline belief formation, whereas I use inflation forecasts. Inflation forecasts in the SPF generally perform better than most forecasting models (Faust and Wright (2013)). This feature of the data will imply agents have better average forecasts of inflation, which may mean the choice of data generates a more conservative role for higher order beliefs. Moreover, since zero coupon yields are constructed based on estimates from prices of different kinds of outstanding Treasury debt, there may be a concern that the "model" concept of Treasury yields is different from the concept that the SPF forecasters had in mind, which might exaggerate deviations of yields from rate expectations. This could influence estimates of the higher order wedge. Furthermore, the quarterly time series for interest rate forecasts in the SPF is much shorter than the inflation forecast data and a worse response rate. Inflation forecasts are available for the whole sample period. The higher order wedge appears to play a greater influence in the Barillas and Nimark (2015) results once rate forecasts become available.

7 What do traders learn from?

In this section, I characterize the informativeness of agents' signals – public and private. Since agents' noisy signals are true on average, it is possible that asset prices effectively filter idiosyncratic noise, and agents are able to determine the true realization of fundamentals. On the other hand, because prices reflect the beliefs of agents, it may be possible that yields do not contain any information that agents do not already know. As discussed above, the estimated results imply that despite abundant common information, agents' information is imperfect and dispersed. By applying techniques from information theory, we can characterize more precisely what information agents draw from what sources.

To preview the results, agents learn about half of what they know about the output and inflation gaps from their private signals, and learn much less about policy or the financial risk factor. The rest of their information about fundamentals comes from prices. The majority of traders' information about the beliefs of others also comes from observing prices. The most important price signal appears to be the policy rate of the central bank, which is also the yield on a bond that matures in one period. The short rate is informative about fundamentals, and because everyone knows that everyone learns about fundamentals from this particular signal, it is also informative about higher-order beliefs.

Information-theoretic concepts In particular, agents' posterior uncertainty can be characterized in terms of entropy, which can be thought of as the average number of binary signals needed to fully describe the outcome of a random variable.[34] We can characterize how much agents learn from signals about a particular variable in terms of the reduction in entropy after observing those signals (see appendix B.7 for details). Adapting a measure used in Melosi (2017), I examine how informed agents are after viewing a counterfactually limited subset of signals *relative* to to how informed they would be if I let them use the complete set. In other words, I calculate how informed they are after observing all of their private signals and the yield curve. I then calculate how informed they would have been under a counterfactual subset of signals. Since on average additional information must (weakly) reduce uncertainty, we can think of the reduction in entropy coming from the subset of signals as the fraction of total information that could have come from that set. If the reduction in entropy were zero, it would imply there was no information in those signals. The advantage of this measure is that it respects both that agents' inference is optimal (by assuming that

[34]The entropy-based measure of signal informativeness I use is also used in the rational inattention literature initiated by Sims (2003) to describe the constraint on agents' information processing capacity.

Table 2: Share of reduction in uncertainty about fundamentals (columns) coming from observing a single signal (rows).

Signal(↓), fundamental →	π	g^y	τ	v
r_t	0.42	0.45	0.97	0.64
π^j	0.03	0.03	0.12	0.00
g^{y^j}	0.03	0.07	0.04	0.00
τ^j	0.00	0.00	0.09	0.00
v^j	0.37	0.33	0.03	0.24

they do the best they can with whatever signals they have) and signals may be redundant.

Table 2 shows the relative reduction in uncertainty about macroeconomic fundamentals variables (columns) from observing the short rate (first row) or a single private signal (remaining rows). These represents extreme constraints on agents' information. The second row, for instance, suggests, that very little (around 3%) of the information traders have about inflation comes from their inflation signal in particular (second row, first column).[35] Three features of the table stand out. First, individual private signals are not terribly informative in general. Second, as to be expected from the fact that agents understand the structure of the model, signals are informative not just about their own realization but about the realizations of other variables; for example, knowing something about inflation tells you something about the output gap. This feature of the world is ignored in most exogenous information models because they typically assume agents learn about independent exogenous processes.

Indeed, observing only the short rate would give you more information about fundamentals than observing any *individual* noisy signal. This is likely for two related reasons. First, the short rate depends directly on the contemporaneous realization of all fundamentals. Second, it is observed without error. Despite the fact that agents are unable to perfectly identify which fundamental moved the short rate, they do know that noise does not factor into their observation; any movement in the short rate is important. Recall also that the table does not imply that observing r_t means that agents observe τ_t nearly perfectly; it only implies that the majority of what limited information they have is encoded in their observation of the central bank's policy instrument.

Table 3 shows (relative to the benchmark with price signals) how much agents' posterior uncertainty is reduced by conditioning only on their four private signals. Here, I switch to considering the risks agents face (i.e., leaving rates and inflation in terms of their gaps)

[35]The columns will not generally sum to 1 because some information is redundant between signals and because yields are also informative.

Table 3: Share of reduction in posterior uncertainty about about fundamentals (top row) and higher order beliefs (subsequetn rows) from observing only private signals

Order of expectation(↓), fundamental (→)	g^r	g^π	g^y	τ	v
x_t	0.02	0.41	0.41	0.16	0.24
$x_t^{(1)}$	0.01	0.09	0.08	0.09	0.02
$x_t^{(2)}$	0.02	0.12	0.11	0.08	0.03
$x_t^{(3)}$	0.04	0.12	0.12	0.08	0.04

rather than realizations.

As the first row of the table reveals, agents' private signals are most informative about the inflation and output gaps. Agents get just under half of their information about the macroeconomy from their private signals. They can learn very little about risk and the implicit inflation target from observing their idiosyncratic signals and almost nothing about the rate *gap* $g_t^r \equiv r_t - \tau_t$ (recall they are assumed to not observe the short rate in this counterfactual). The remaining lines show how much of their information about (higher-order) expectations come from private signals. Since private signals are about the true realization of variables, rather than higher-order beliefs about those variables, they are only indirect signals about higher order beliefs.

Another way of thinking about the results in table 3 is "what are price signals informative about?" It turns out that the majority of information agents have about the financial risk factor (v) and monetary policy (summarized by g^r and τ) comes from observing price signals, including the short rate. Nearly all of their information about the first three orders of expectations is encoded in price signals (rows 2-4). Yield curve variables may not be fully informative about fundamentals or the beliefs of others, but the vast majority of information traders have about the latter seems to come from prices.

This result has two immediate implications. First, it validates thinking of the yield curve as a summary measure of what bond traders believe. Indeed, the model implies that the best *bond traders* can do to understand what others believe is by combining their understanding of how expectations are determined with the prices they observe. Since prices depend mostly on higher-order beliefs, prices are useful to bond traders even though they aren't fully informative about fundamentals. Second, the results caution against ignoring the informativeness of prices for modeling inference. Agents may have very inaccurate private signals on average, but the ability to learn from prices makes that less consequential. This matters directly for models featuring dispersed information. An econometrician calibrating the informativeness of private signals using only forecasts while ignoring the role of learning

Table 4: Reduction in posterior uncertainty about fundamentals and higher order beliefs from observing private signals and r_t

Order of expectation(↓), fundamental (→)	g^r	g^π	g^y	τ	v
x_t	0.98	0.85	0.88	0.99	0.93
$x_t^{(1)}$	0.86	0.72	0.72	0.92	0.90
$x_t^{(2)}$	0.85	0.81	0.78	0.90	0.92
$x_t^{(3)}$	0.86	0.85	0.84	0.90	0.94

from prices might incorrectly conclude that private signals must be quite accurate.

The estimates imply most of what traders learn comes from combining of their private signals and the short rate. The results of this counterfactual are shown in table 4. Effectively all of what they know about monetary policy and risk comes from their private signals plus the policy rate, and 75% or more of what they know about the first three orders of expectation can be extracted without using bonds with a maturity of greater than one quarter.

This result adds to recent evidence (Nakamura and Steinsson (2018), Tang (2013) Melosi (2017)) that the Federal Reserve's policy instrument is an important signal, at least over the sample period. It tells market participants a great deal about macroeconomic fundamentals and policy risks. Because it is an informative *public* signal, it also plays an outsized role in market participants' higher order beliefs (along the lines of Morris and Shin (2002)). Assuming agents learn only from private signals and the policy rate – the information assumption used in Melosi (2017) and Kohlhas (2015) – is a fair approximation of what bond traders appear to learn from.

To emphasize the fact that the policy rate is somewhat special, table 5 shows a counterfactual where agents learn from their private signals and ten year yields rather than the federal funds rate. Ten year yields are informative about both fundamentals and higher-order beliefs above and beyond private signals, but they are less informative than the policy rate. This is especially true about the current policy gap (the first column) and the risk factor v (the last column).

Why are ten years yields less informative? The price of a ten year bond is determined not just by fundamentals (the short rate), but also higher order beliefs about the evolution of fundamentals over the next ten years, plus the maturity-specific shock. The fact that the bond is of longer maturity means that (increasingly) higher order beliefs play a greater role in its price. The fact that shocks to fundamentals are transitory, higher-order beliefs play a bigger role in prices, and that bond prices are affected by maturity specific shocks, imply they will be less informative about current fundamentals. If, for instance, bond prices were

Table 5: Reduction in posterior uncertainty about fundamentals and higher order beliefs from observing private signals and ten year yield

Order of expectation(↓), fundamental (→)	g^r	g^π	g^y	τ	v
x_t	0.23	0.82	0.76	0.94	0.60
$x_t^{(1)}$	0.10	0.68	0.55	0.70	0.36
$x_t^{(2)}$	0.12	0.77	0.65	0.66	0.32
$x_t^{(3)}$	0.17	0.82	0.72	0.63	0.30

not affected by noise, then they would be fully revealing of fundamentals and agents would not need to rely on the short rate as a source of information.

These results come with a few caveats. First, in keeping with the majority of the literature, the model is constructed specifically to price a single type of asset. Other types of assets may be informative about a different set of macroeconomic or idiosyncratic risks. However, if other asset prices are informative for agents about fundamentals, that would be be captured by the precision of private signals. Second, the results of this section are partial equilibrium in the sense that the model does not allow for direct feedback from expectations to macroeconomic aggregates. But from the point of view of an atomistic agent, macroeconomic aggregates are exogenous processes and the precise role of information in generating aggregate fluctuations should not matter.

8 Conclusion

Survey evidence suggests professional forecasters have dispersed beliefs about future prices of Treasury bonds and macroeconomic variables. In this paper, I construct and estimate a structural model that reflects this feature of the world. My estimates imply that the direct role of belief dispersion is modest, but that most of the time variation in the higher order wedge is caused by policy-related factors. In particular, the wedge grew during the 1970s and early 1980s, along with the central bank's implicit inflation target, and fell over the course of the Great Moderation. This is consistent with gradual learning by agents about a new monetary policy regime and the emergence of a consensus about the conduct of monetary policy. A natural catalyst for this was greater transparency and credibility of central bankers.

The results add to the body of evidence that deviations from full information are an important feature of the world. Accounting for agents' inference dramatically affects the size and interpretation of term premia, even without constraints on using prices as information or assuming traders have model inconsistent beliefs. The result that asset prices appear to be an important source of macroeconomic information suggests general equilibrium macroeconomic models with dispersed information should account for learning from prices when quantifying the importance of these frictions or when assessing normative questions. It also suggests, at least for asset prices, market consistent information is not enough for aggregate irrelevance of information frictions. This is true in two senses: Dispersed information directly affects prices and the behavior of endogenous beliefs is quite different than under full information.

In this paper, I have focused on the informational content of a single type of asset - nominal government debt. Other assets may have different information implications. Extending the analysis to debt of different countries – along the lines of Wright (2011) – may also be informative about how changes in the monetary policy framework are associated with changes in the importance of higher-order beliefs. Throughout the paper I have taken advantage of the fact that yields are affine. This makes characterizing the higher order wedge and informativeness of signals straightforward. However, in the aftermath of the 2008 financial crisis and the 2020 COVID-19 pandemic, there were nonlinearities in yields introduced by the zero lower bound. These may have affected the information content of prices. Connolly and Struby (2020) explore the implications of various common-information term structure models during the Great Recession and recovery, but do not incorporate dispersed information for tractability reasons; this avenue remains open for future work.

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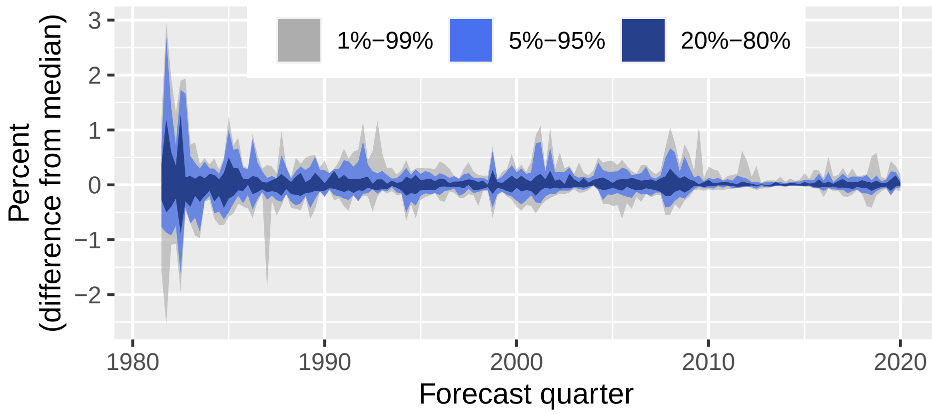
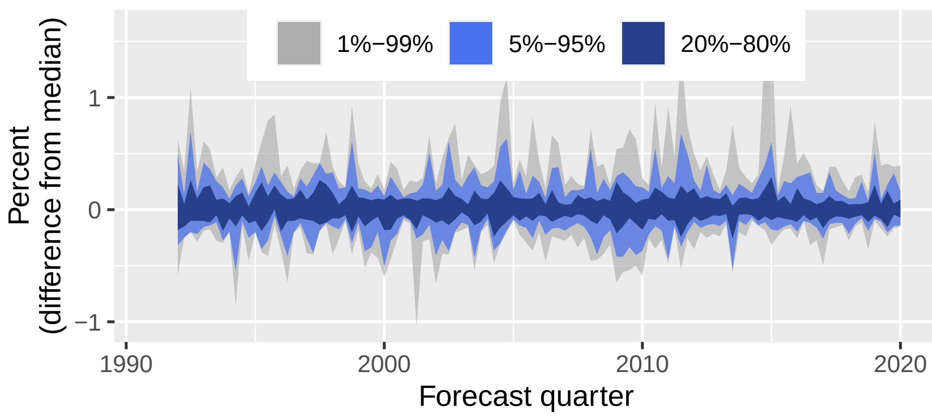
Figures



(a) Dispersion about the median nowcast, Treasury bills



(b) Dispersion about the median nowcast, Treasury bonds

Figure 1: Dispersion of current-quarter forecasts for Treasury yields from the SPF. For each quarter, the median forecast is subtracted. Bands correspond to the 1-99%, 5-95%, and 20-80% percentile ranges of nowcasts.

Rate shock: Macroeconomic Responses

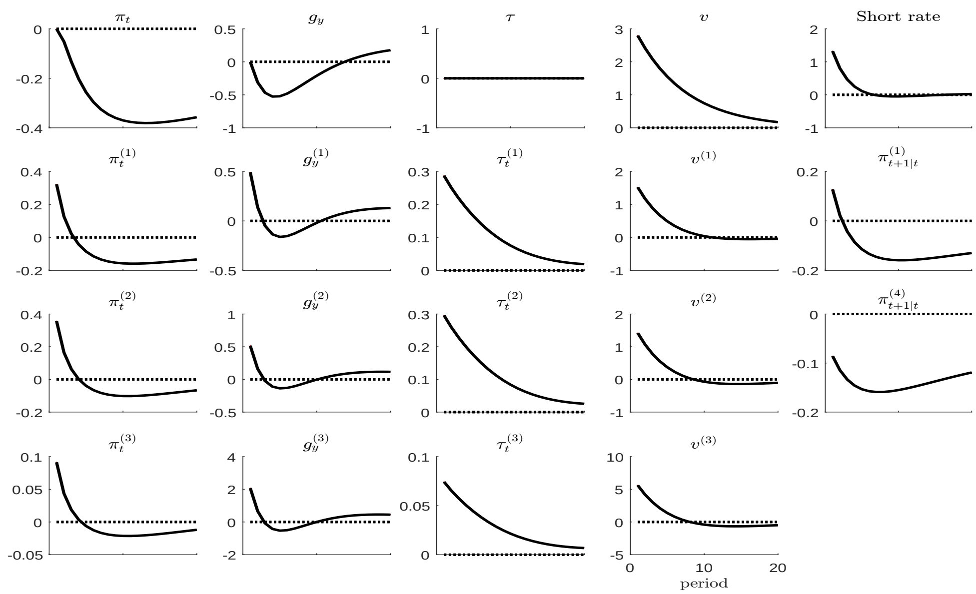


Figure 2: Response of non-financial variables to monetary policy rule shock, dispersed information model. The top row shows realized fundamentals. The first four columns of subsequent rows show average beliefs about fundamentals, second order beliefs, and third order beliefs, for five years following the fundamental shock.

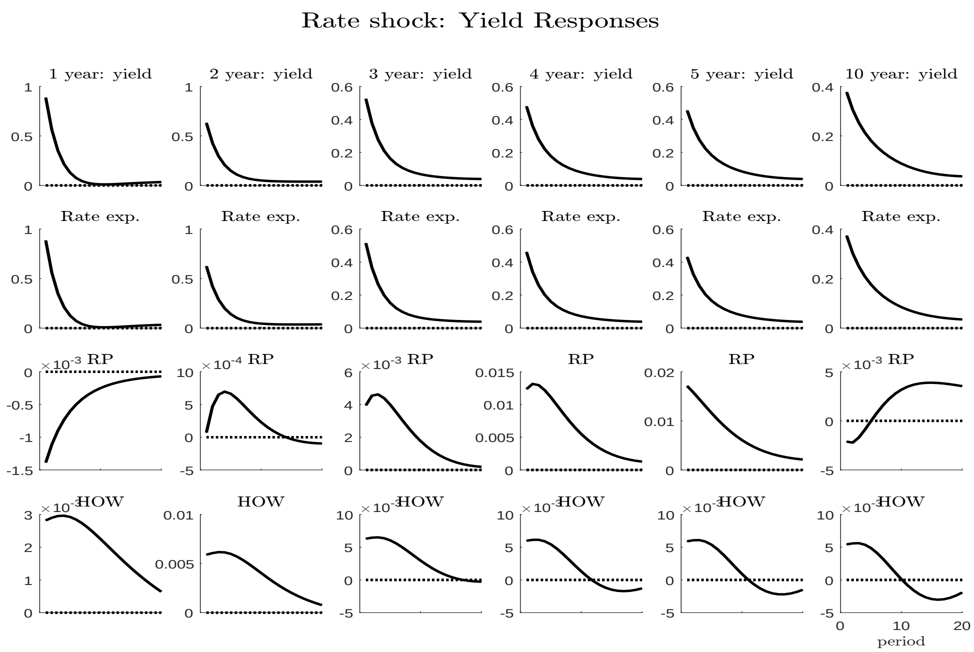


Figure 3: Response of financial variables to monetary policy rule shock, dispersed information. The top row shows the percentage change in yields. Subsequent rows decompose it into the contribution of interest rate expectations, classical risk premia (RP), and the higher order wedge (HOW).

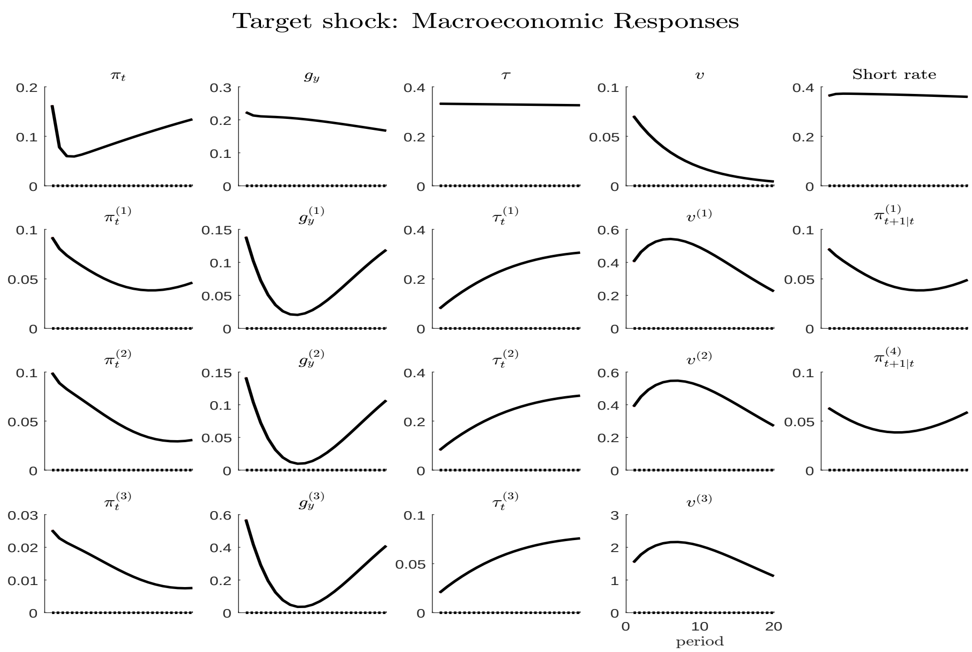


Figure 4: Response of non-financial variables to inflation target (τ) shock, dispersed information

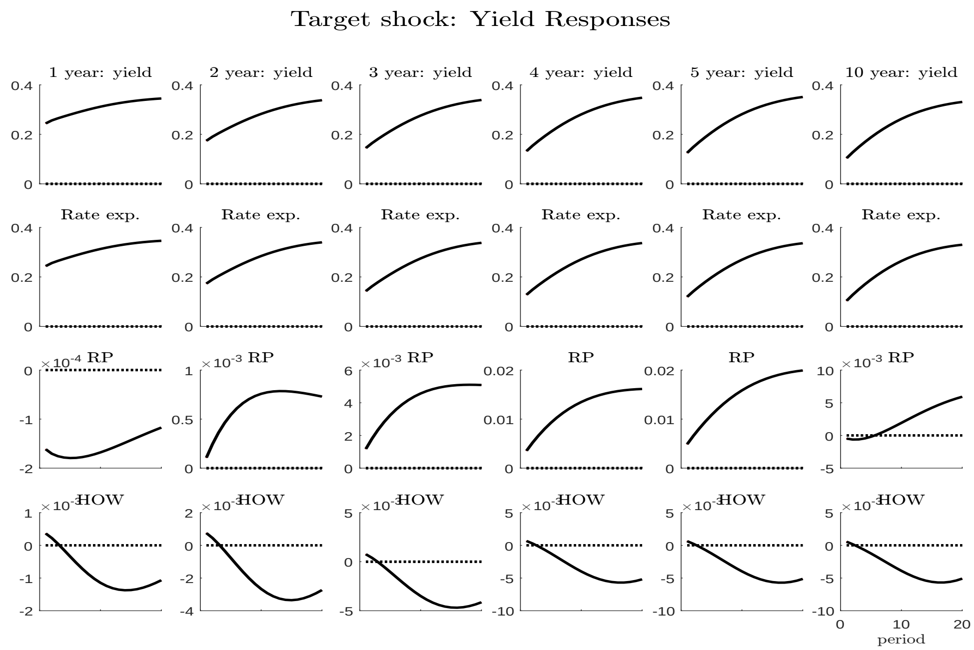


Figure 5: Response of financial variables to inflation target shock, dispersed information

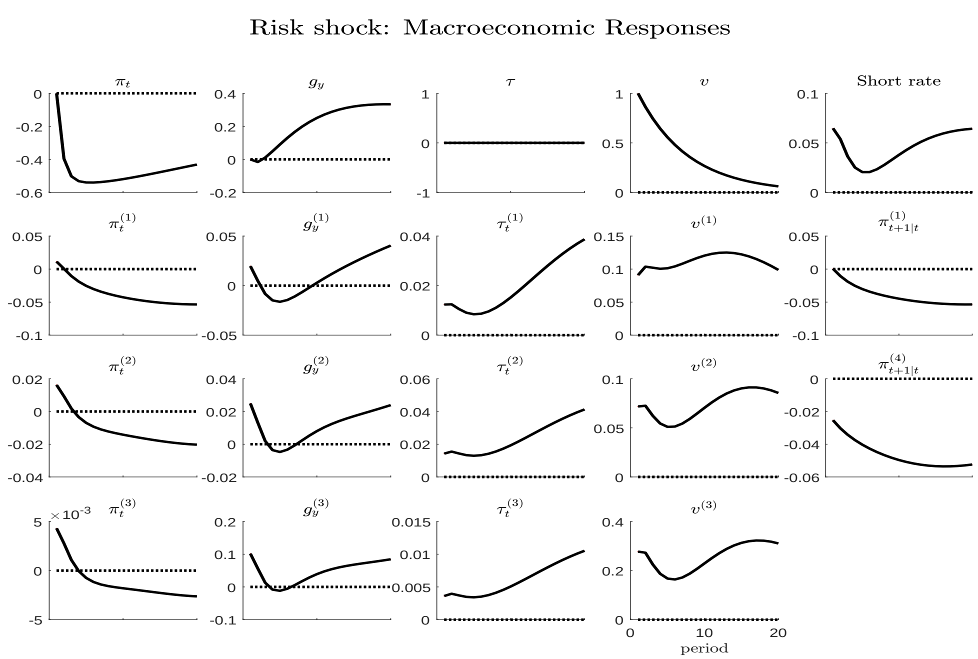


Figure 6: Response of non-financial variables to risk shock, dispersed information

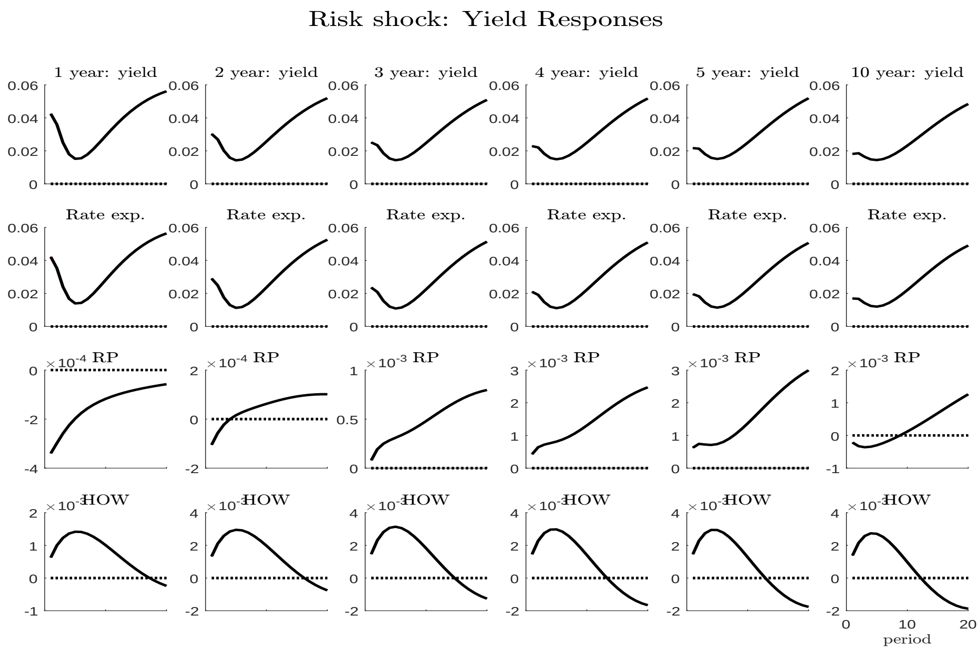


Figure 7: Response of financial variables to risk shock, dispersed information

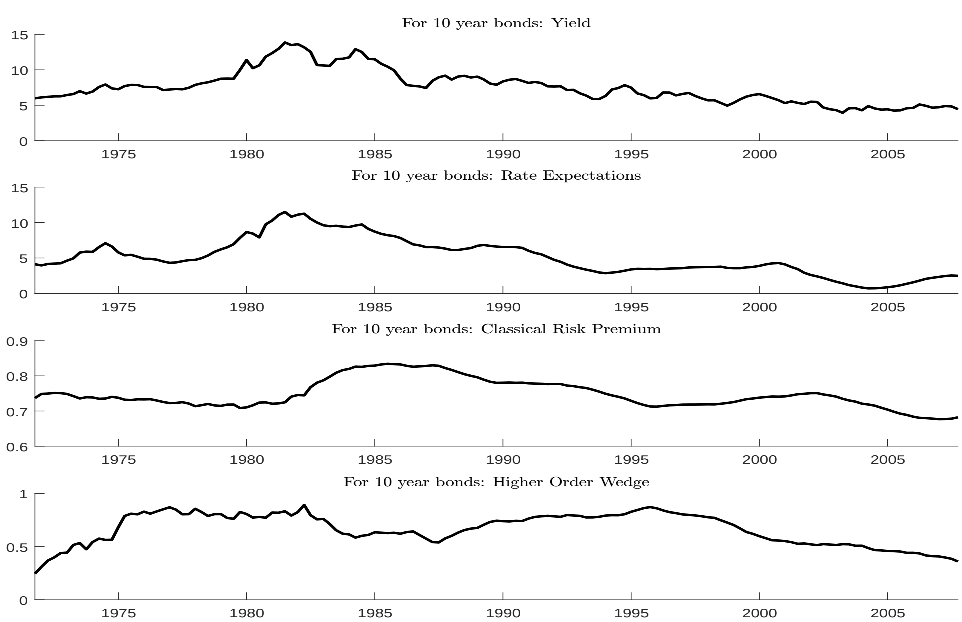


Figure 8: Decomposition of 10 year yields, dispersed information

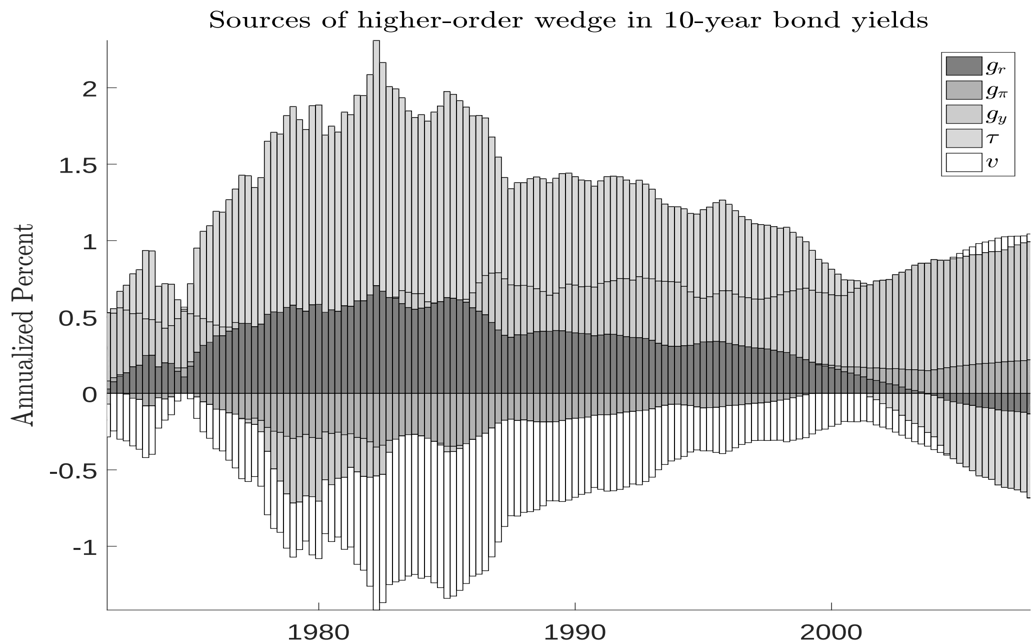


Figure 9: Decomposition of higher order wedge, ten year yields. Details of the decomposition are found in section B.6.

A Reduced form evidence for information rigidities

In this appendix, show the details of the methodology of Coibion and Gorodnichenko (2015) to interest rate forecasts as discussed in the main text, and show the complete regression results.

For simplicity, assume Treasury bill rates follow an AR(1) process but agents observe idiosyncratic, noisy signals about the realization of that process. Innovations and signal noise are assumed to be normally distributed and mean zero:

$$\begin{aligned} r_t = & \quad \rho r_{t-1} + \varepsilon_t \text{ with } \rho \in [0,1) \\ r_{it} = & \quad r_t + e_{it} \end{aligned}$$

Assuming agents are Bayesian learners, their conditional expectations can be written as:

$$\begin{aligned} E_t^i r_t = \kappa r_{it} + (1-\kappa) E_{t-1}^i r_t \\ E_t^i r_{t+h} = \rho^h E_t^i r_t \end{aligned}$$

Their expectation of the short rate is a weighted average of their current signal and their prior, where κ is the relative weight placed on the signal. As with the notation in the main model, I use $r^{(1)}_{t|t}$ to indicate the average expectation of r_t at time t.

Averaging across agents and rearranging gives the relationship between the forecast error for the *average* forecast and the revision of the average forecast at each horizon h:

$$r_{t+h} - r^{(1)}_{t+h|t} = \frac{1-\kappa}{\kappa}\left(r^{(1)}_{t+h|t} - r^{(1)}_{t+h|t-1}\right) + \sum_{j=1}^{h} \rho^{h-j}\varepsilon_{t+h} \tag{24}$$

where the error term is the sum of rational expectations errors. If signals were perfectly informative, $\kappa = 1$, and there would be no weight on forecast revisions in this regression. To the extent agents face information frictions, $\kappa < 1$. The simple reduced-form test of information frictions in financial forecasts amounts to projecting forecast revisions on forecast errors; the null hypothesis of full information rational expectations is equivalent to testing whether the regression coefficient is 0. Finding a significant positive coefficient, on the other hand, suggests information frictions. The regression takes the form of (1) in the main text.

The results of conducting this for different forecast horizons are shown in table 6 for 3-month Treasury bills. The results for bills are broadly consistent with Coibion and Gorodnichenko (2015)'s findings for inflation. The estimated coefficient is positive and significant, suggesting average forecasts for financial variables reflect dispersed information among individuals. Treasury bond results are shown in table 7. Here, the results are somewhat less

Table 6: Results of regression (1) for Treasury bill forecasts in SPF, 1981:Q3 - 2020:Q1

	Dependent variable:							
	h = 1		h = 2		h = 3		h=4	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Forecast Revision for h	0.170***	0.191***						
	(0.023)	(0.023)						
			0.225***	0.293***				
			(0.077)	(0.075)				
					0.339***	0.498***		
					(0.128)	(0.127)		
							0.616***	0.878***
							(0.173)	(0.171)
Constant	−0.039***		−0.129***		−0.277***		−0.400***	
	(0.011)		(0.044)		(0.072)		(0.092)	
R^2	0.272	0.317	0.054	0.092	0.045	0.093	0.079	0.151
Note: HAC standard errors in parentheses								*p<0.1; **p<0.05; ***p<0.01

clear; over one period horizons, the regression coefficient is positive and significant, while for other horizons the significance is mixed but the coefficient is generally positive, consistent with information frictions. Moreover, the constant in the regression is significant, suggesting forecast errors are, in fact, predictable on average – also inconsistent with FIRE.

Table 7: Results of regression (1) for Treasury bond forecasts in SPF, 1992:Q2 - 2020:Q1

	Dependent variable:							
	h = 1		h = 2		h = 3		h=4	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Forecast Revision for h	0.158***	0.204***						
	(0.039)	(0.038)						
			0.054	0.231*				
			(0.116)	(0.118)				
					−0.089	0.228		
					(0.161)	(0.174)		
							−0.066	0.397*
							(0.198)	(0.222)
Constant	−0.056***		−0.219***		−0.381***		−0.513***	
	(0.016)		(0.049)		(0.066)		(0.076)	
R^2	0.131	0.206	0.002	0.034	0.003	0.016	0.001	0.029
Note:								*p<0.1; **p<0.05; ***p<0.01

B Model derivations

B.1 Intuition

Beginning with

$$P_t^n = E_t^j \left[M_{t+1}^j P_{t+1}^{n-1} \right]$$

Joint lognormality implies:

$$p_t^n = E_t^j[m_{t+1}^j] + E_t^j[p_{t+1}^{n-1}] + \frac{1}{2}\text{Var}(m_{t+1}^j + p_{t+1}^{n-1})$$

Iterating ahead for another agent (an arbitrary k that agent j will sell the bond to)

$$p_{t+1}^{n-1} = E_{t+1}^k[m_{t+2}] + E_{t+1}^k[p_{t+2}^{n-2}] + \frac{1}{2}\text{Var}(m_{t+2}^k + p_{t+2}^{n-2})$$

Then substituting this into the price expectation term:

$$\begin{aligned} p_t^n &= E_t^j[m_{t+1}^j] \\ &+ E_t^j[E_{t+1}^k(m_{t+2}^k)] + E_t^j[E_{t+1}^k p_{t+1}^{n-2}] + E_t^j[E_{t+1}^k p_{t+1}^{n-2}] \\ &+ \frac{1}{2}\text{Var}(m_{t+1}^j + p_{t+1}^{n-1}) + \frac{1}{2}\text{Var}(m_{t+2}^k + p_{t+2}^{n-2}) \end{aligned}$$

The fact that information sets are not nested means the law of iterated expectations does not apply. However, because no agent has particular information about other agents, agent j's expectations about k's expectations can be replaced by her expectation of the average expectation. Doing so, and integrating both sides over all agents implies the equation in the text.

B.2 The filtering problem

The individual agent's filtering problem, and its aggregation into the vector of average higher order expectations, follows Nimark (2007) and Barillas and Nimark (2015).

Call X_t the underlying state we want to estimate (the vector of higher order expectations, including 0th order expectations). Call $\Sigma_{t|t-1} \equiv E[(X_t - X_{t|t-1})(X_t - X_{t|t-1})']$.

Forecast step. Given information dated time $t-1$, j's forecast of the signal is

$$z_{t|t-1}^j = \mu_Z + DX_{t|t-1} \tag{25}$$

The associated covariance matrix of signal forecasting error is

$$\begin{aligned}\Omega_{t|t-1} &\equiv E[(z_t^j - z_{t|t-1}^j)(z_t^j - z_{t|t-1}^j)'] \\ &= D\Sigma_{t|t-1}D' + RR'\end{aligned} \tag{26}$$

Updating step. Projection of $X_t - X_{t|t-1}$ onto $z_t^j - z_{t|t-1}^j$ and rearrangement gives that j's conditional expectation of the state given her time t information is

$$\begin{aligned}X_{t|t}^j &= X_{t|t-1} + \underbrace{\Sigma_{t|t-1}D'\Omega_{t|t-1}^{-1}}_{K_t}(z_t^j - z_{t|t-1}^j) \\ &= X_{t|t-1}^j + K(DX_t + R\begin{bmatrix}u_t \\ \eta_t^j\end{bmatrix} - DX_{t|t-1}^j) \\ &= \mu_X + \mathcal{F}X_{t-1|t-1}^j + K[D(\mu_X + \mathcal{F}X_{t-1} + \mathcal{C}u_t) + R\begin{bmatrix}u_t \\ \eta_t^j\end{bmatrix} - D(\mu_X + \mathcal{F}X_{t-1|t-1}^j)]\end{aligned} \tag{27}$$

Deriving the aggregate law of motion. Partition R into a part associated with aggregate shocks and one associated with idiosyncratic shocks, i.e. $R \equiv \begin{bmatrix}R_u & R_\eta\end{bmatrix}'$. Integrating $X_{t|t}^j$ to obtain the vector of *average* higher order expectations "zeros out" the idiosyncratic shocks:

$$X_{t|t} = \mu_X + (\mathcal{F} - KD\mathcal{F})\, X_{t-1|t-1} + KD\mathcal{F}X_{t-1} + K(D\mathcal{C} + R_u)u_t \tag{28}$$

These expressions have been written in terms of the steady state Kalman gain K, which is derived using the discrete-time algebraic Riccati equation

$$\begin{aligned}\Sigma_{t+1|t} &= E[(X_{t+1} - X_{t+1|t})(X_{t+1} - X_{t+1|t})'] \\ &= \mathcal{F}(\Sigma_{t|t-1} - \Sigma_{t|t-1}D'\Omega_{t|t-1}^{-1}D\Sigma_{t|t-1})\mathcal{F}' + RR'\end{aligned} \tag{29}$$

The resulting steady state $\Sigma_{t+1|t}$, combined with (26), immediately implies K. Matching coefficients between (28) and the conjected VAR process for X_t yields:

$$\begin{aligned}\mathcal{F} &= \begin{bmatrix}F^P & \mathbf{0}_{d\times d\bar{k}} \\ \mathbf{0}_{d\bar{k}\times d} & \mathbf{0}_{d\times d\bar{k}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{d\times d} & \mathbf{0}_{d\times d\bar{k}} \\ \mathbf{0}_{d\bar{k}\times d} & [\mathcal{F} - KD\mathcal{F}]_{\_}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{d\times d(\bar{k}+1)} \\ [KD\mathcal{F}]_{\_}\end{bmatrix} \\ \mathcal{C} &= \begin{bmatrix}C & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{0} \\ [K(D\mathcal{C} + R_u)]_{\_}\end{bmatrix}\end{aligned} \tag{30}$$

where $_{\_}$ indicates truncation to ensure conformability and truncation at $\bar{k}$.

B.3 Generating bond price equations

The steps here are identical to Barillas and Nimark (2015). Starting with the conjected price expression:

$$p_t^n = A_n + B_n' X_t + \nu_t^n$$

To arrive at this form, substitute the SDF (10) into the (log) arbitrage condition:

$$p_t^n = \ln E\left[\exp\left\{-r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - \Lambda_t^{j\prime}a_{t+1}^j + p_{t+1}^{n-1}\right\}|\Omega_t^j\right] \tag{31}$$

Here we use the definition of a_{t+1}^j (11) to substitute p_{t+1}^{n-1} out for its expectation plus the forecast error for that particular maturity

$$P_{t+1}^{n-1} = E\left[p_{t+1}^{n-1}|\Omega_t^j\right] + e_{n-1}' a_{t+1}^j \tag{32}$$

where e_n' is a horizontal selection vector with 1 in the nth element and zeros elsewhere.

Rationality implies:

$$E[p_{t+1}^{n-1}|\Omega_t^j] = A_{n-1} + B_{n-1}' \underbrace{\left(\mu_X + \mathcal{F}E[X_t|\Omega_t^j]\right)}_{E[X_{t+1}|\Omega_t^j]} \tag{33}$$

Define an operator H that selects just the average higher order expectations from X_t^j (13), that is

$$\begin{aligned} E[X_t|\Omega_t^j] =& HX_t^j \\ \text{where } H =& \begin{bmatrix} \mathbf{0}_{d\bar{k}\times d} & I_{d\bar{k}} \\ \mathbf{0}_{d\times d} & \mathbf{0}_{d\times d\bar{k}} \end{bmatrix} \end{aligned} \tag{34}$$

Combining these three expressions gives

$$E[p_{t+1}^n|\Omega_t^j] = A_{n-1} + B_{n-1}'\mu_X + B_{n-1}'\mathcal{F}HX_t^j \tag{35}$$

substituting this in to the no-arbitrage condition

$$p_t^n = \ln E\left[\exp\left\{-r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - \Lambda_t^{j\prime}a_{t+1}^j + A_{n-1} + B_{n-1}'\mu_X + B_{n-1}'\mathcal{F}HX_t^j + e_{n-1}'a_{t+1}^j\right\}|\Omega_t^j\right] \tag{36}$$

The inner expression consists of constants and lognormal random variables. It can be written in terms of things known to agent j at time t (so the expectation is superfluous):

$$p_t^n = \ln\exp\Big\{ - r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - A_{n-1} + B'_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t^j + \frac{1}{2}(e'_{n-1}\Sigma_a e_{n-1} + \Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - 2e'_{n-1}\Sigma_a\Lambda_t^j)\Big\} \tag{37}$$

where the last term is 1/2 times the variance of $(e'_{n-1} - \Lambda_t^j)a_{t+1}^j$. Simplifying:

$$p_t^n = -r_t + A_{n-1} + B_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t^j + \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\Lambda_t^j \tag{38}$$

The price of the n period bond at time t is a function of constants, the current risk-free rate, and j specific terms. Like Barillas and Nimark (2015), I focus on a hypothetical agent whose state coincides with the cross-sectional average state. Then we can substitute X_t for X_t^j in the previous expression, since $X_t \equiv \int X_t^j dj$.

Finally substitute (3) and (12) into the previous expression:

$$\begin{aligned} p_t^n =& - (\delta_0 + \delta'_x x_t) + A_{n-1} + B_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t \\ &+ \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\left(\Lambda_0 + \Lambda_x^j + \Lambda_\nu\int E[\nu_t|\Omega_t^j]dj\right) \end{aligned} \tag{39}$$

Define $\delta'_X \equiv \begin{bmatrix}\delta'_x & \mathbf{0}\end{bmatrix}$ and rearrange this

$$\begin{aligned} p_t^n =& - \delta_0 + A_{n-1} + B_{n-1}\mu_X + \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\Lambda_0 \\ &- \delta'_X X_t + B'_{n-1}\mathcal{F}HX_t - e'_{n-1}\Sigma_a\Lambda_x X_t \\ &- e'_{n-1}\Sigma_a\Lambda_\nu\int E[\nu_t|\Omega_t^j]dj \end{aligned} \tag{40}$$

We had guessed

$$p_t^n = A_n + B'_n X_t + \nu_t^n \tag{6}$$

To arrive at the conjectured form, impose two additional restrictions. First, restrict:

$$\Lambda_\nu = -\Sigma_a^{-1} \tag{41}$$

which also reduces the number of free parameters in the model. Secondly, we can substitute to replace the remaining $e'_{n-1}\int E[\nu_t|\Omega_t^j]dj$ term via a convenient normalization. Note model consistent expectations and the conjectured bond price equation imply

$$p_t^n = E[A_n + B_n X_t + \nu_t^n|\Omega_t^j] = A_n + B_n HX_t^j + e'_{n-1}E[\nu_t|\Omega_t^j] \tag{42}$$

Setting this equal to the conjectured bond equation implies

$$\begin{aligned} A_n + B_n H X_t + e'_{n-1}\int E[v_t|\Omega_t^j]dj = & \quad A_n + B_n X_t + \nu_t^n \\ \Rightarrow e'_{n-1}\int E[v_t|\Omega_t^j]dj = & \quad B_n(I-H)X_t + \nu_t^n \end{aligned} \tag{43}$$

Substituting these restrictions:

$$\begin{aligned} p_t^n = & -\delta_0 + A_{n-1} + B'_{n-1}\mu_X + \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\Lambda_0 \\ & -\delta'_X X_t + B'_{n-1}\mathcal{F}HX_t - e'_{n-1}\Sigma_a\Lambda_x X_t \\ & + B_n(I-H)X_t + \nu_t^n \end{aligned} \tag{44}$$

Finally, write $B = \begin{bmatrix} B'_2 & \cdots & B'_{\bar{n}} \end{bmatrix}$ and note that $B_n = e_{n-1}B$. Normalizing prices of risk:

$$\Lambda_x = \widehat{\Lambda}_x + B(I-H) \tag{45}$$

and then

$$\begin{aligned} p_t^n = & -\delta_0 + A_{n-1} + B'_{n-1}\mu_X + \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\Lambda_0 \\ & -\delta'_X X_t + B'_{n-1}\mathcal{F}HX_t - e'_{n-1}\Sigma_a\widehat{\Lambda}_x X_t + \nu_t^n \end{aligned} \tag{46}$$

This implies the recursive forms for the bond price equations in the paper.

B.4 Macroeconomic structure

$$P_0 = \begin{bmatrix} 1 & -(1-\phi_r)\phi_\pi & -(1-\phi_r)\phi_y & 0 & -(1-\phi_r)\phi_v \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mu_x = \begin{bmatrix} (1-\phi_r)g^r - (1-\phi_r)g_y \\ -\rho_{\pi r}g_r - \rho_{\pi y}g^y \\ g_y - \rho_{yr}g^r - \rho_{yy}g^y \\ (1-\rho_{\tau\tau})\tau \\ 0 \end{bmatrix} \tag{47}$$

$$P_1 = \begin{bmatrix} \phi_r & 0 & 0 & 0 & 0 \\ \rho_{\pi r} & \rho_{\pi\pi} & \rho_{\pi y} & 0 & \rho_{\pi v} \\ \rho_{yr} & \rho_{y\pi} & \rho_{yy} & 0 & \rho_{yv} \\ 0 & 0 & 0 & \rho_{\tau\tau} & 0 \\ 0 & 0 & 0 & 0 & \rho_{vv} \end{bmatrix}, \quad \Sigma_0 = \begin{bmatrix} \sigma_r & 0 & 0 & 0 & 0 \\ 0 & \sigma_\pi & 0 & \sigma_{\pi\tau}\sigma_\tau & 0 \\ 0 & \sigma_{y\pi}\sigma_\pi & \sigma_{y\pi}\sigma_\pi & \sigma_{y\tau}\sigma_\tau & 0 \\ 0 & 0 & 0 & \sigma_\tau & 0 \\ \sigma_{vr} & \sigma_{v\pi} & \sigma_{vy} & \sigma_{v\tau} & \sigma_v \end{bmatrix} \tag{48}$$

$$\delta_0 = \mathbf{0}_{5\times 1}, \quad \delta'_x = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{49}$$

and the matrices governing the evolution of fundamentals (4)

$$\mu^P = P_0^{-1}\mu_0, \quad F^P = P_0^{-1}P_1, \quad C = P_0^{-1}\Sigma_0 \tag{50}$$

B.5 Restrictions on Prices of Risk

To impose the Ireland (2015) restrictions on the stochastic discoutn factor (10) and prices of risk (12), I set:

$$\lambda_0 = \begin{bmatrix}\lambda^r & \lambda^\pi & \lambda^y & \lambda^\tau & 0\end{bmatrix}', \quad \lambda_x = \begin{bmatrix} 0 & 0 & 0 & 0 & \lambda_x^r \\ 0 & 0 & 0 & 0 & \lambda_x^\pi \\ 0 & 0 & 0 & 0 & \lambda_x^y \\ 0 & 0 & 0 & 0 & \lambda_x^\tau \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{51}$$

To additionally impose the Barillas and Nimark (2015) restriction, recall that the vector of bond price innovations a_{t+1}^j is a linear combination of forecasting error in the factors X_{t+1} and maturity-specific price shocks ν_{t+1}.

$$a_{t+1}^j = \Psi \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{52}$$

To see this, write j's one-period ahead bond pricing error for a particular maturity as

$$\begin{aligned} a_{t+1}^{n,j} &= p_{t+1}^{n-1} - p_{t+1|t}^j \\ &= B_{n-1}'(X_{t+1} - E_t^j X_t) + \nu_{t+1}^{n-1} \end{aligned} \tag{53}$$

So stacking these errors in a vector a_{t+1}^j gives

$$a_{t+1}^j = \underbrace{\begin{bmatrix} B_1' & \\ \vdots & I_{\bar{n}-1} \\ B_{\bar{n}-1}' & \end{bmatrix}}_{\Psi} \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{54}$$

Left multiplying by $\Lambda_{t+1}^{j\prime}$:

$$\Lambda_{t+1}^{j\prime} a_{t+1}^j = \Lambda_{t+1}^{j\prime}\Psi \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{55}$$

We want to restrict this so that

$$\Lambda_{t+1}^{j\prime} a_{t+1}^{j} = \left(\begin{bmatrix} \lambda_0 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \lambda_x & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right)' \begin{bmatrix} X_{t+1} - E\left[X_{t+1} | \Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{56}$$

If we removed dispersed information or maturity-specific shocks, this restriction would imply only fundamentals matter for bond prices, given the restrictions in (51) and (**??**). When maturity specific shocks are equal to zero, these additional restrictions must hold:

$$\begin{aligned} \begin{bmatrix} \lambda_0 \\ \mathbf{0} \end{bmatrix}' \begin{bmatrix} X_{t+1} - E\left[X_{t+1} | \Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} &= \Lambda_0' a_{t+1}^j \\ \left(\begin{bmatrix} \lambda_x & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} X_t^j \right)' \begin{bmatrix} X_{t+1} - E\left[X_{t+1} | \Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} &= \widehat{\Lambda}_x' a_{t+1}^j \end{aligned} \tag{57}$$

where $\widehat{\Lambda_x}$ is a normalization (see appendix B.3). This can be achieved by setting

$$\begin{aligned} \Phi = \Psi(\Psi'\Psi)^{-1}, \qquad & \Lambda_0 = \Phi \begin{bmatrix} \lambda_0 \\ \mathbf{0} \end{bmatrix} \\ \widehat{\Lambda}_x = \Phi \begin{bmatrix} \lambda_x & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad & \Psi = \begin{bmatrix} B_1' & \\ \vdots & I_{\bar{n}-1} \\ B_{\bar{n}-1}' & \end{bmatrix} \end{aligned} \tag{58}$$

These restrictions are the same as those imposed in Barillas and Nimark (2015).

B.6 Bond price decompositions

Given the expression for prices and a model for inference, we can characterize what portion of bond yields are driven by higher-order beliefs - that is, the portion of yields driven directly by dispersed information. Common knowledge of rationality and fact that X_t has a Markov structure implies (1) bond prices are pinned down by the *current* state and thus agents' forecasts of *future* states determine their forecasts of future bond prices, and (2) all information about future X_t is summarized in today's state (Barillas and Nimark (2015)). Hence, two agents who agree about X_t agree about X_{t+1}, X_{t+2}, etc, and thus agree about price forecasts. Intuitively, the difference between actual prices and the price that would obtain if all agents counterfactually held the same beliefs is the direct contribution of dispersed information to the bond price.[36] Like Barillas and Nimark (2015), I use the wedge between

[36] Allen et al. (2006) show in a similar setting how prices of long-lived assets will not generally reflect average expectations when there is private information. Barillas and Nimark (2015) refer to the difference between actual prices and the counterfactual consensus price as the "speculative component"; Bacchetta and Van Wincoop (2006) refer to it as the "higher order wedge." The preferred interpretation of Bacchetta and Van Wincoop is that it is the present value of deviations of higher-order beliefs from first-order beliefs.

the counterfactual price with common beliefs and actual prices to quantify the extent to which dispersed information about particular factors affects bond yields. Moreover, because priced risks have a macroeconomic interpretation, the wedge can be decomposed in order to understand whether disagreement about particular macroeconomic conditions are important for determining yields at different maturities.

Define a matrix operator $\bar{H}$ that replaces all higher order expectations with first order expectations, that is:

$$\begin{bmatrix} x_t & x_t^{(1)} & \vdots & x_t^{(1)} \end{bmatrix}' = \bar{H} \begin{bmatrix} x_t & x_t^{(1)} & \vdots & x_t^{(\bar{k})} \end{bmatrix}' \tag{59}$$

The price that would obtain if all higher order expectations coincided with the first order expectation - the "counterfactual consensus price" - is

$$\bar{p}_t^n = A_n + B_n' \bar{H} X_t + \nu_t^n \tag{60}$$

We can use this to decompose prices into the component that depends on averag (first order) expectations and the component that depends on dispersion of information and the resulting divergence of expectations about expectations. The wedge can be written as:

$$p_t^n - \bar{p}_t^n = B_n' X_t - B_n' \bar{H} X_t = B_n'(I - \bar{H}) X_t \tag{61}$$

The counterfactual consensus price, which contains only the effect of average expectations in yields, can be decomposed into short rate expectations and "classical" risk premia - that is, the part of yields that depends on first-order average beliefs net of average rate expectations.

$$p_t^n = A_n^{\text{prem}} + B_n^{\text{prem}\prime} X_t + A_n^{\text{rate}} + B_n^{\text{rate}\prime} X_t + \underbrace{B_n'(I - \bar{H}) X_t}_{\text{higher order wedge}} + \nu_t^n \tag{62}$$

Where $A_n^{\text{prem}} = A_n - A_n^{\text{rate}}, B_n^{\text{prem}\prime} = B_n' \bar{H} - B_n^{\text{rate}\prime}$. To make this decomposition operative, we need the model-implied future expected short rates. For the hypothetical average agent,

$$E_{t|t} r_{t+1} = -\delta_0 - \delta_X H X_{t+1|t} = -\delta_0 - \delta_X (\mu_X + \mathcal{F} H X_t)$$

and so on for further ahead future short rates:

$$
\begin{aligned}
A_n^{\text{rate}} &= -\, n(\delta_0 + \delta_X \mu_X) - \delta_X \sum_{s=0}^{n-1} \mathcal{F}^s \mu_X \\
B_n^{\text{rate}\prime} &= -\, \delta_X \sum_{s=0}^{n-1} \mathcal{F}^s H
\end{aligned}
\tag{63}
$$

The decomposition of the wedge is a straightforward selection of different elements. For example, the portion of the higher-order wedge attributable to higher-order beliefs about the long-run inflation target τ_t is

$$
B_n'(I - \bar{H})X_t^{\tau} \equiv \quad B_n'(I - \bar{H}) \cdot \text{diag}\begin{bmatrix}0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 0\end{bmatrix} X_t \tag{64}
$$

Note that this depends on both the level of the (higher order) expectations (i.e., $\tau_t^{(2)}, \tau_t^{(3)}$) and so on), and how that level translates into compensation for risk (from B_n').

B.7 Information-theoretic concepts

In section 7, I refer to a number of concepts from information theory. More details of these concepts are found in Veldkamp (2011) and Cover and Thomas (2006). I characterize the extent to which variables are informative using the notion of entropy – the amount of information required to describe a random variable (Cover and Thomas (2006)). Intuitively, the entropy of a random variable in log_2 units (bits) is the average number of binary signals required to describe its realization.

The entropy of a normally distributed variable. If x is a normally distributed variable with variance σ^2, its entropy is $\frac{1}{2}\log_2(2\pi e \sigma^2)$ (Cover and Thomas, 2006, Chapter 8).

Conditional entropy. Conditional entropy $H(x|y)$ is a measure of how much information it takes to describe x given that y is known (Veldkamp, 2011, Chapter 3.2). It is defined as the joint entropy of x, y minus the entropy of y, that is $H(x|y) = H(x, y) - H(y)$. The calculation of the conditional entropy of a normal variable is analogous to the unconditional case, replacing the variance with the conditional variance (Veldkamp (2011)).

Mutual information. The mutual information of two variables x and y, $I(x; y)$ is the measure of the amount of information one contains about the other. It can be calculated in terms of entropies ((Cover and Thomas, 2006, Theorem 2.4.1))):

$$I(x, y) = H(x) - H(x|y) = H(y) - H(y|x)$$

Measure of signal use. Similar to Melosi (2014, 2017), I use the "share" of mutual information to characterize the amount of information that comes from (a set of) of signals ω_{red}. In particular, the "share" of information about a variable x used by an agent is:

$$Share_x = I(x; \omega_{red})/I(x; \omega_{full)}$$

where ω_{red} is the reduced set of signals (for example, only private signals without the use of bond prices) and ω_{full} is the complete set of signals available.

In practice, conditional variances needed to calculate mutual information are taken as particular entries from agents' state nowcasting error matrix ($\Sigma_{t|t}$) (see appendix B.2). The conditional variance of the subset of signals is calculated by solving the filtering problem of the agent assuming they have a "counterfactual" subset of signals (just as described in appendix B.2, using $A, B, \mathcal{F}, \mathcal{C}$ from the actual model solution. This share is bounded between 0 and 1 because, on average, conditioning must reduce entropy (Cover and Thomas, 2006, Theorem 2.6.5).

C Estimation procedure and results

C.1 Econometric matrices

The model-consistent notion of dispersion of signals around the average comes from the idiosyncratic part of agents' Kalman filtering problem. The idiosyncratic error covariance matrix is the solution to the following Riccati equation:

$$\begin{aligned} \Sigma_j &= E[(X_t^j - X_t^{(1)})(X_t^j - X_t^{(1)})'] \\ &= (\mathcal{F} - KD\mathcal{F})\Sigma_j(\mathcal{F} - KD\mathcal{F})' + KR_\eta R_\eta' K' \end{aligned} \tag{65}$$

Hence the cross-sectional variance in average forecasts is just the appropriate element of Σ_j:

$$Var(\pi^j_{t|t}) = \underbrace{[0, 1, 0, 1, 0, \mathbf{0}_{1\times d*(\bar{k})]}]}_{\equiv e^\pi} \Sigma_j e^{\pi\prime} \tag{66}$$

For the full information model, the equations are the same. the observed bond yields are assumed to be observed with yield-specific error,and the cross-sectional estimation error terms for the forecasts are replaced with horizon-specific error terms $\widetilde{\sigma_\pi}^h, h = 1, 4$.

C.2 Priors

Table 8: Prior distribution of model parameters

Parameter	Prior distribution	Prior mean	Prior s.d.	Model
ϕ_r	Beta	0.5000	0.0500	
ϕ_π	Gamma	0.5000	0.3000	
ϕ_y	Gamma	0.5000	0.3000	
ϕ_v	Trunc. Normal	0.0000	0.5000	
σ_r	Inverse Gamma	0.0050	0.2000	
ρ_{yr}	Normal	-1.0000	0.5000	
$\rho_{y\pi}$	Normal	0.0000	0.5000	
ρ_{yy}	Inverse Gamma	0.9000	0.2000	
ρ_{yv}	Trunc. Normal	0.0000	2.0000	
$\sigma_{y\pi}$	Normal	0.0000	1.0000	
$\sigma_{y\tau}$	Normal	0.0000	1.0000	
σ_y	Inverse Gamma	0.1000	3.0000	
σ_τ	Inverse Gamma	0.0050	0.3000	
$\rho_{\pi r}$	Normal	0.0000	2.0000	
$\rho_{\pi\pi}$	Inverse Gamma	0.9000	0.2000	
$\rho_{\pi y}$	Normal	0.0000	0.5000	
$\rho_{\pi v}$	Normal	0.0000	2.0000	
$\sigma_{\pi\tau}$	Normal	0.0000	3.0000	
σ_π	Inverse Gamma	0.0050	0.3000	
ρ_{vv}	Beta	0.8000	0.1000	
σ_{vr}	Normal	0.0000	3.0000	
$\sigma_{v\pi}$	Normal	0.0000	3.0000	
σ_{vy}	Normal	0.0000	3.0000	
$\sigma_{v\tau}$	Normal	0.0000	3.0000	
λ_r	Uniform(-100,100)			
λ_π	Uniform(-100,100)			
λ_y	Uniform(-100,100)			
λ_τ	Uniform(-100,100)			
λ_r^x	Uniform(-100,100)			
λ_π^x	Uniform(-100,-0.001)			
λ_y^x	Uniform(-100,100)			
λ_τ^x	Uniform(-100,100)			
σ_ν	Uniform(0.001,0.02)			DI
$\widetilde{\sigma_\pi}$	Uniform(0.001,100)			DI
$\widetilde{\sigma_y}$	Uniform(0.001,100)			DI
$\widetilde{\sigma_\tau}$	Uniform(0.001,100)			DI
$\widetilde{\sigma_v}$	Uniform(0.001,100)			DI
$\widetilde{\sigma}_4$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_8$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{12}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{16}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{20}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{40}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_\pi^1$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_\pi^4$	Inverse Gamma	1.0000	3.0000	FI

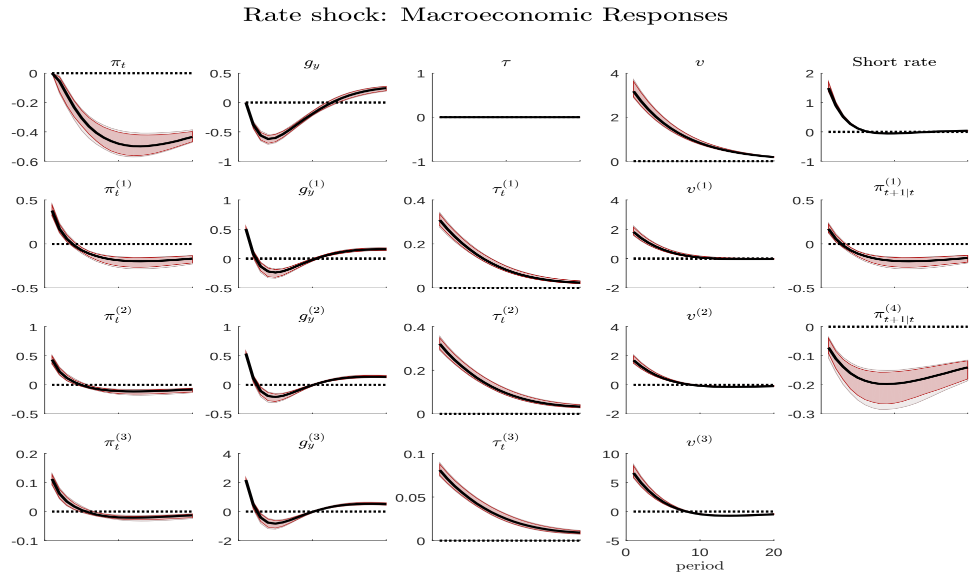
C.3 Dispersed-information parameter results

	Mode	Mean	Median	5%	95%	Std.
ϕ_r	0.5357	0.5339	0.5355	0.5324	0.5370	0.0224
ϕ_π	0.1835	0.1851	0.1869	0.1768	0.1908	0.0090
ϕ_y	0.1016	0.1010	0.1010	0.0928	0.1101	0.0066
ϕ_v	0.0349	0.0343	0.0348	0.0305	0.0365	0.0024
σ_r	0.0029	0.0033	0.0032	0.0029	0.0038	0.0003
ρ_{yr}	-0.9329	-0.9954	-0.9994	-1.0440	-0.9312	0.0539
$\rho_{y\pi}$	-0.3871	-0.4088	-0.4134	-0.4361	-0.3730	0.0254
ρ_{yy}	0.9164	0.8988	0.9031	0.8794	0.9136	0.0389
ρ_{yv}	-0.0007	-0.0008	-0.0006	-0.0022	-0.0000	0.0007
$\sigma_{y\pi}$	0.3329	0.2873	0.2711	0.2443	0.3355	0.0362
$\sigma_{y\tau}$	2.6888	2.6788	2.6878	2.6491	2.7163	0.1140
σ_y	0.0062	0.0068	0.0068	0.0063	0.0071	0.0004
σ_τ	0.0008	0.0009	0.0009	0.0008	0.0010	0.0001
$\rho_{\pi r}$	0.9229	0.8949	0.8882	0.8650	0.9323	0.0453
$\rho_{\pi\pi}$	0.4331	0.4368	0.4372	0.4223	0.4564	0.0208
$\rho_{\pi y}$	-0.1998	-0.1870	-0.1872	-0.2027	-0.1724	0.0117
$\rho_{\pi v}$	-0.1139	-0.1105	-0.1114	-0.1143	-0.1062	0.0055
$\sigma_{\pi\tau}$	-0.1380	-0.1483	-0.1452	-0.1993	-0.1123	0.0309
σ_π	0.0031	0.0034	0.0033	0.0031	0.0037	0.0002
ρ_{vv}	0.8633	0.8600	0.8613	0.8589	0.8640	0.0360
σ_{vr}	9.7831	9.8318	9.8834	9.7764	9.8987	0.4138
$\sigma_{v\pi}$	2.1485	2.1574	2.1672	2.1123	2.1919	0.0932
σ_{vy}	-2.1067	-2.1144	-2.1111	-2.1939	-2.0638	0.0972
$\sigma_{v\tau}$	0.8502	0.8581	0.8577	0.8215	0.9125	0.0466
λ_r	1.3216	1.0637	1.1132	0.7289	1.3235	0.1872
λ_π	-4.6156	-4.6789	-4.6655	-4.9244	-4.4789	0.2289
λ_y	0.0223	-0.5522	-0.5930	-1.0610	-0.0260	0.3227
λ_τ	-0.1725	-0.2202	-0.1318	-0.7908	0.1247	0.2885
λ_r^x	18.0439	18.3595	18.5122	17.9321	18.8519	0.8392
λ_π^x	-76.2162	-76.1999	-76.2163	-76.6236	-76.1171	3.1893
λ_y^x	-17.4621	-17.4327	-17.4515	-17.9745	-17.0402	0.7816
λ_τ^x	0.0126	0.1011	0.0987	-0.0640	0.3192	0.1232
σ_ν	0.0025	0.0032	0.0031	0.0025	0.0041	0.0005
$\widetilde{\sigma}_\pi$	0.3418	0.4817	0.5082	0.3303	0.6348	0.1106
$\widetilde{\sigma}_y$	0.7511	0.6247	0.6539	0.4601	0.7715	0.1073
$\widetilde{\sigma}_\tau$	0.5445	0.5860	0.5825	0.4727	0.7032	0.0849
$\widetilde{\sigma}_v$	0.4116	0.4153	0.4175	0.3039	0.5054	0.0609

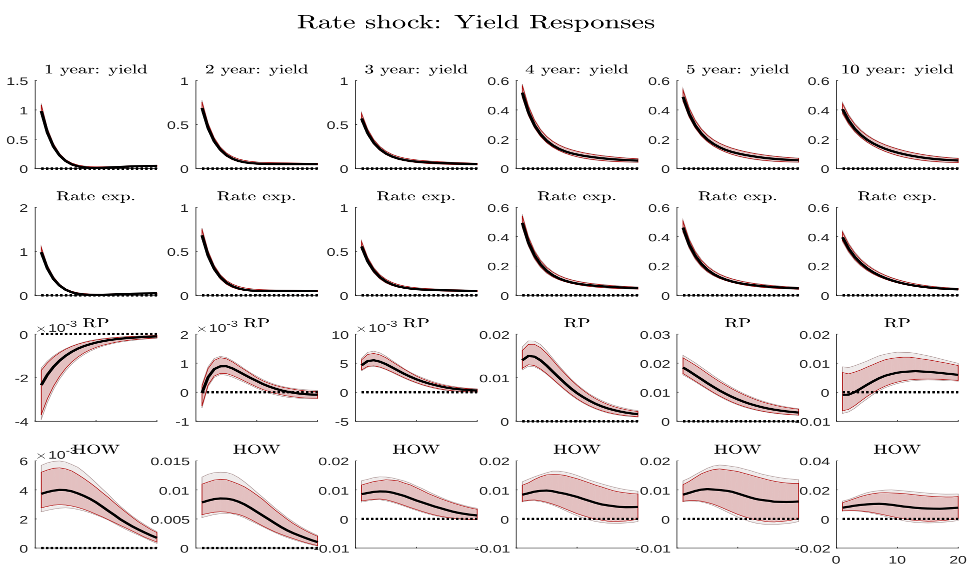
Table 9: Parameter estimates for dispersed information model

C.4 Additional Results, dispersed information model

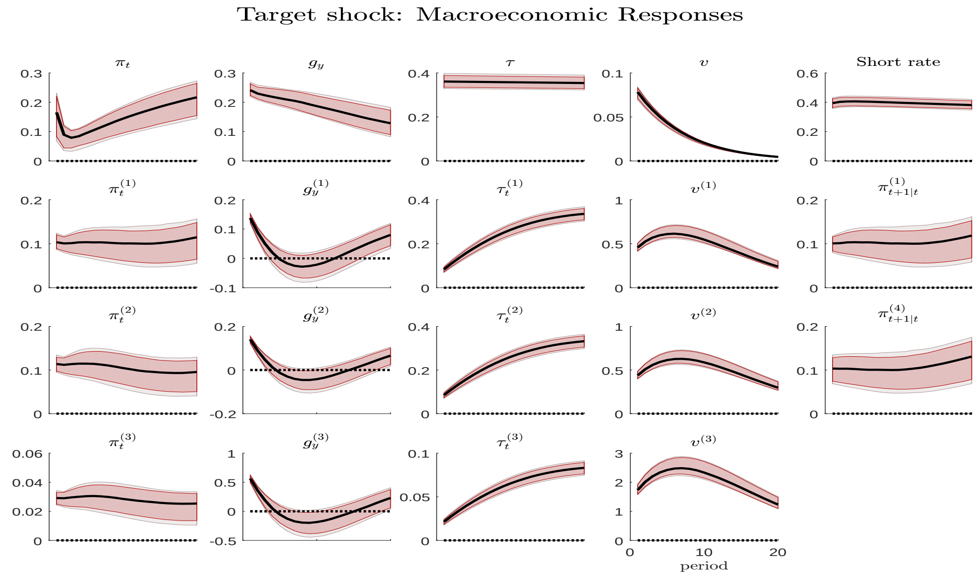
C.4.1 Impulse Responses



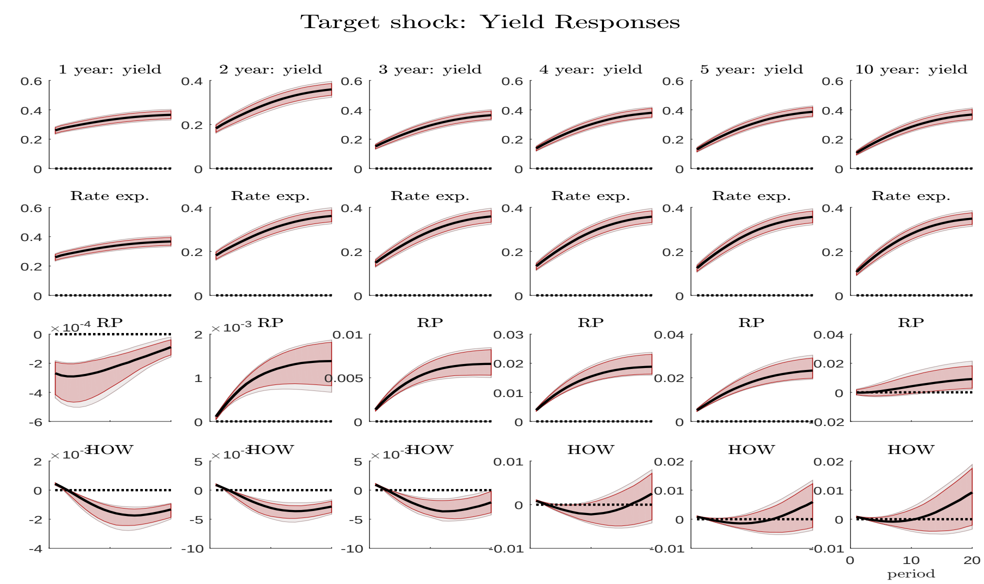
(a) Response of non-financial variables to monetary policy rule shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.



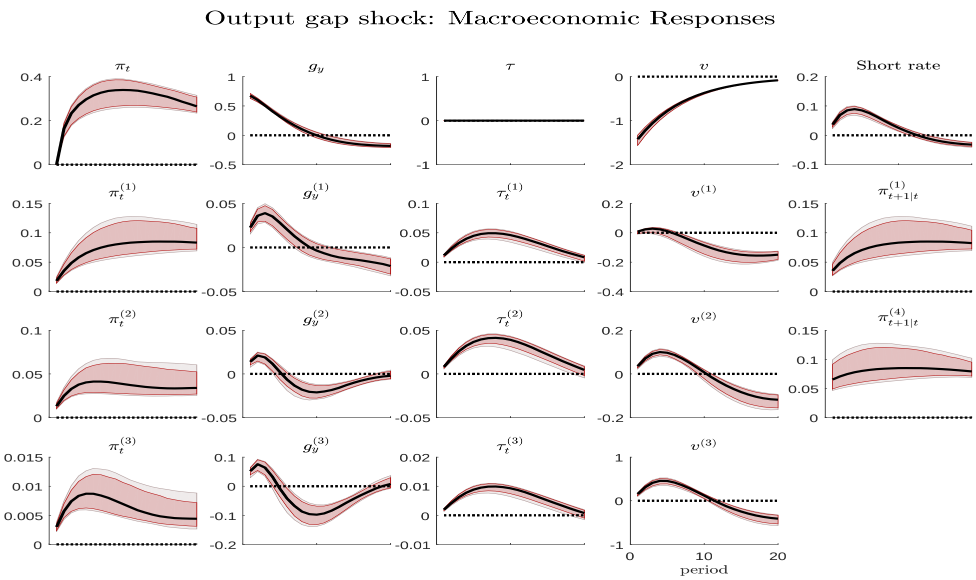
(b) Response of financial variables to monetary policy rule shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.



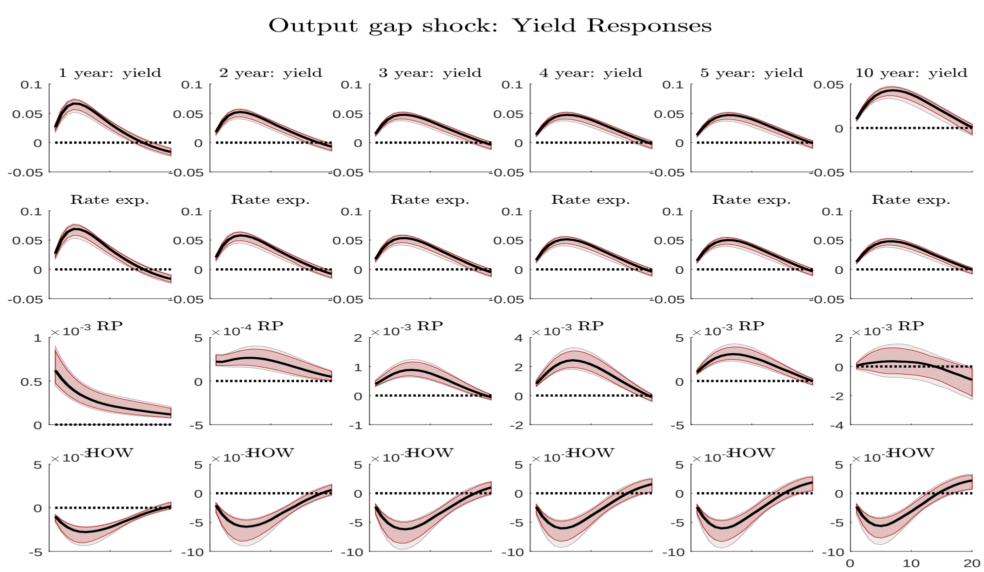
(a) Response of non-financial variables to inflation target shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. υ is scaled by 100.



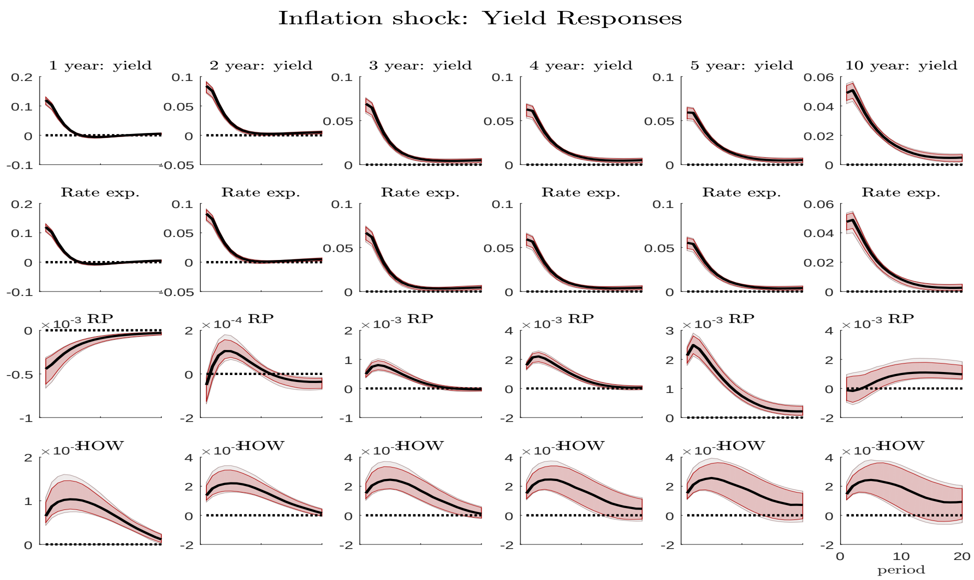
(b) Response of financial variables to inflation target shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. υ is scaled by 100.



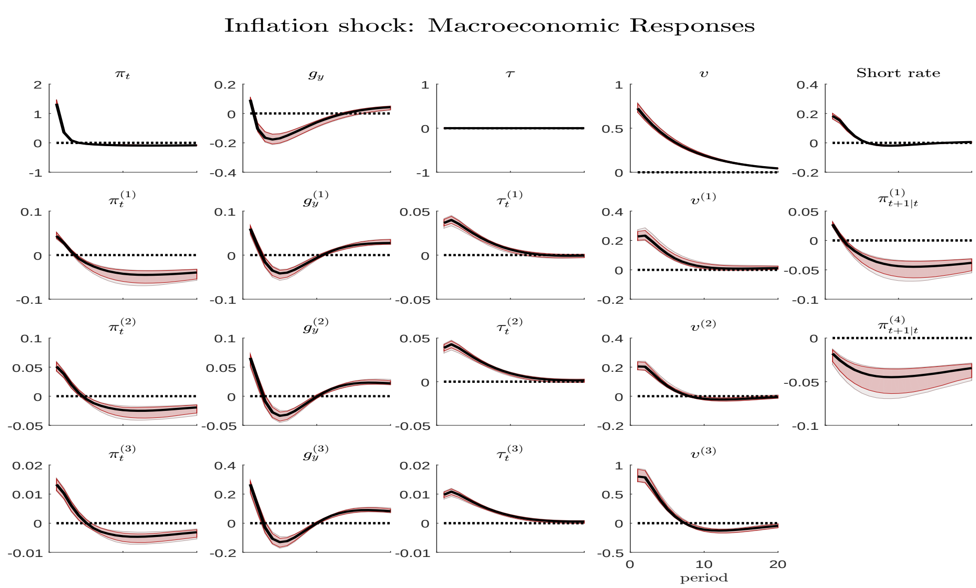
(a) Response of non-financial variables to output gap shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. υ is scaled by 100.



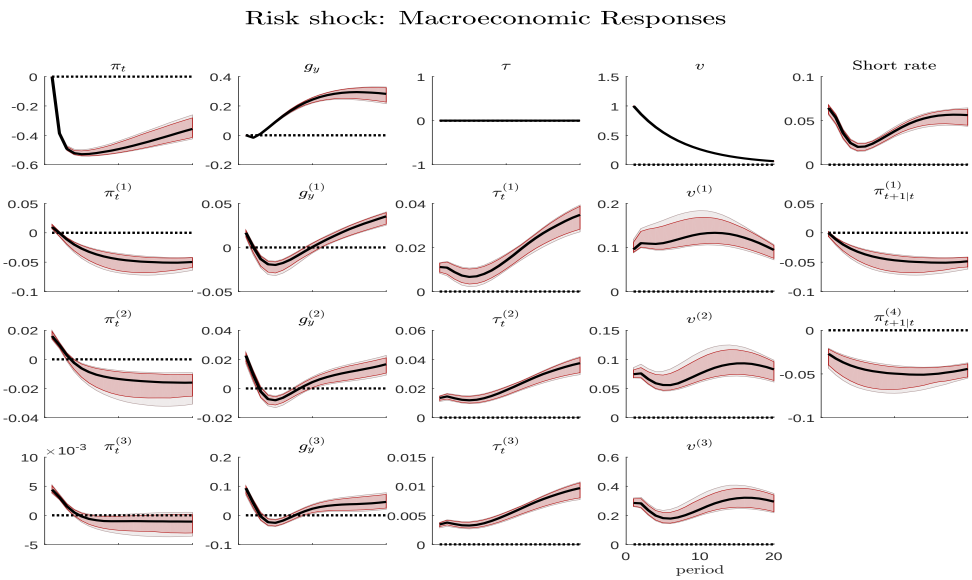
(b) Response of financial variables to output gap shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Interest rates are shown in annualized percentage points.



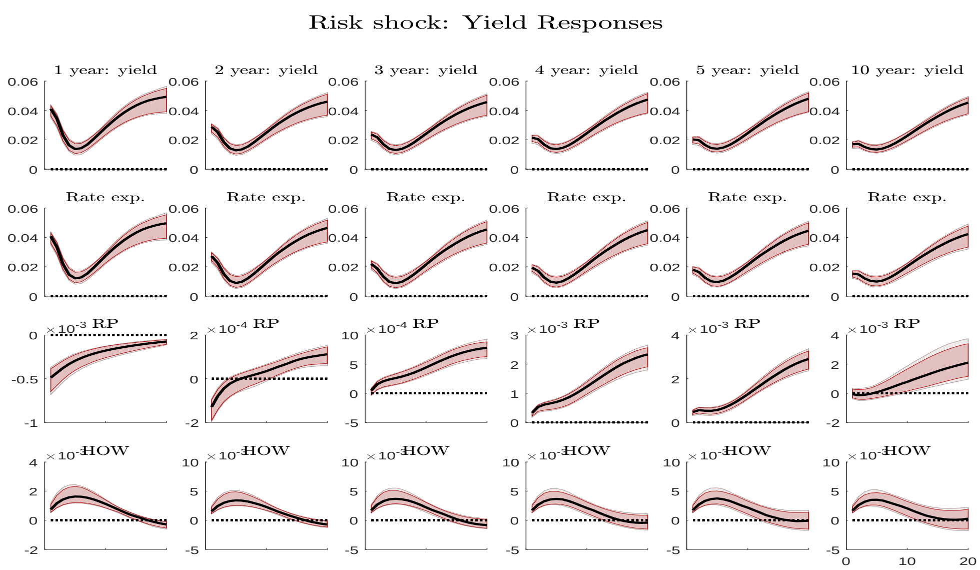
(a) Response of non-financial variables to inflation shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.



(b) Response of financial variables to inflation shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Interest rates are shown in annualized percentage points.



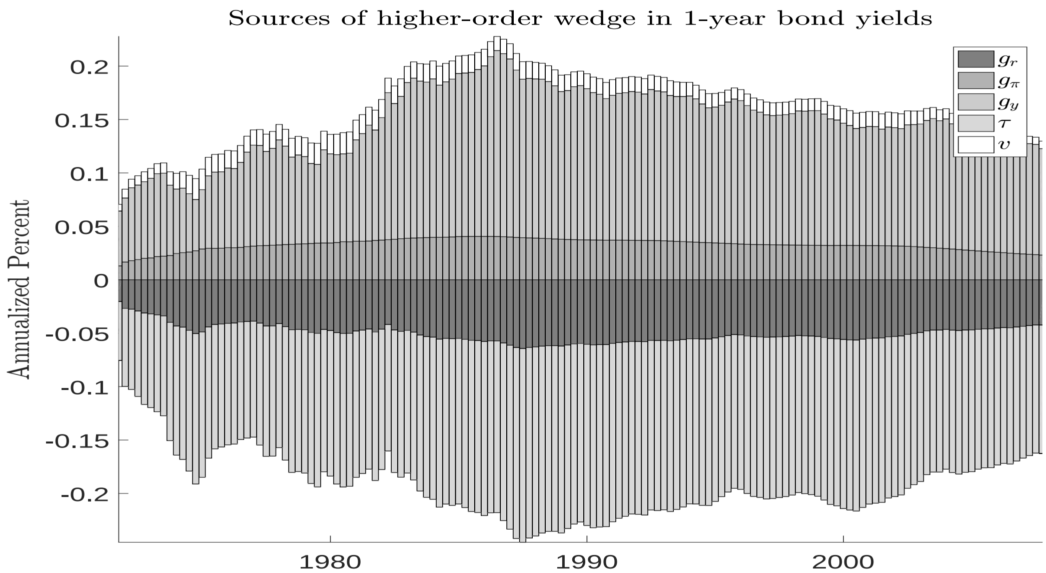
(a) Response of non-financial variables to risk shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.

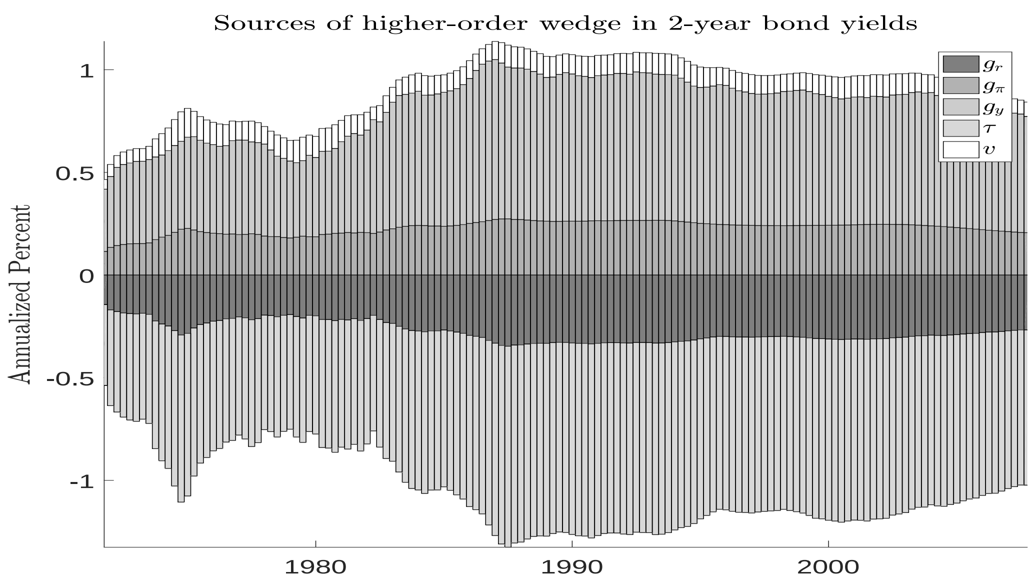


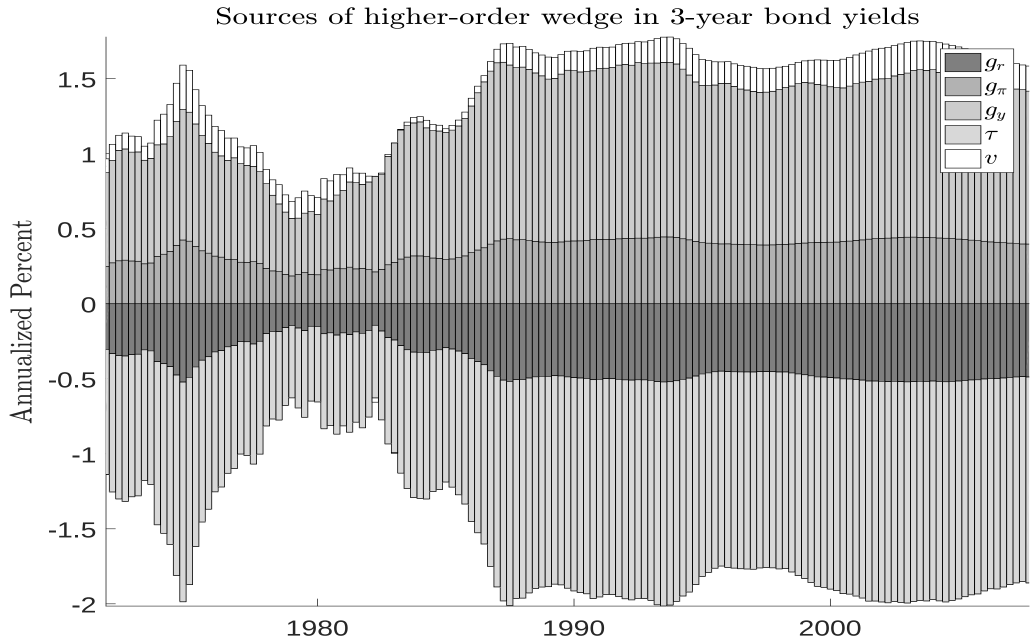
(b) Response of financial variables to risk shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Interest rates are shown in annualized percentage points.

D Additional results

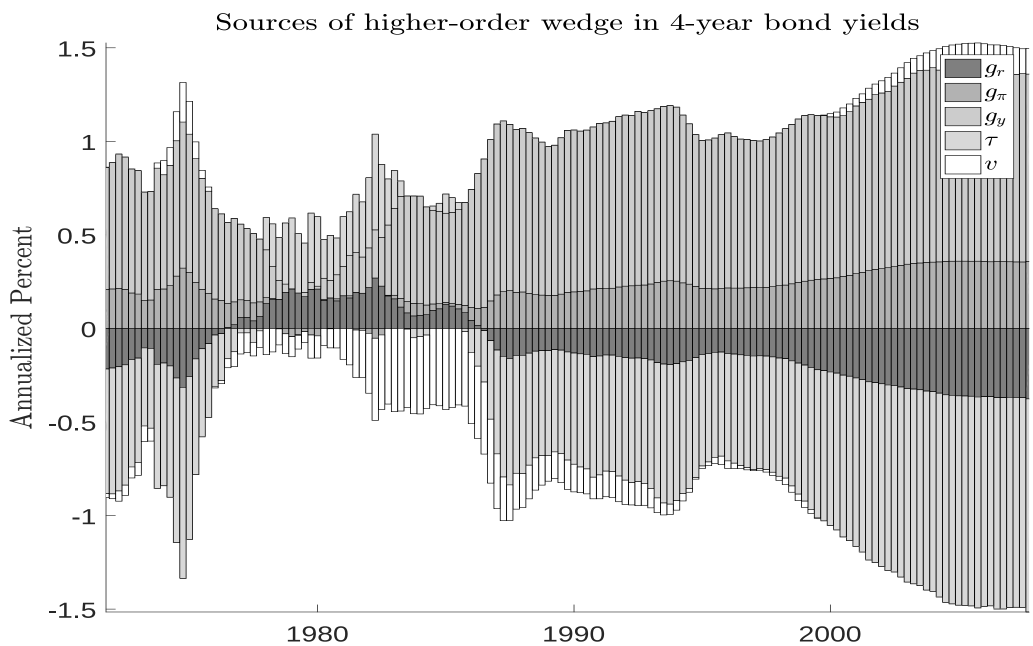
D.1 Wedge Decompositions at Posterior Mode



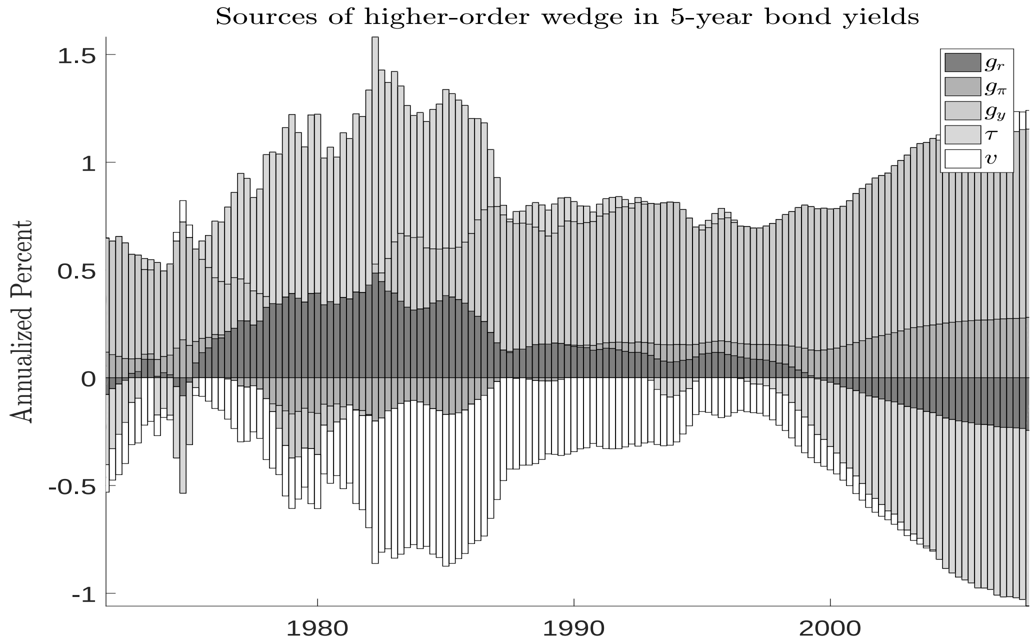




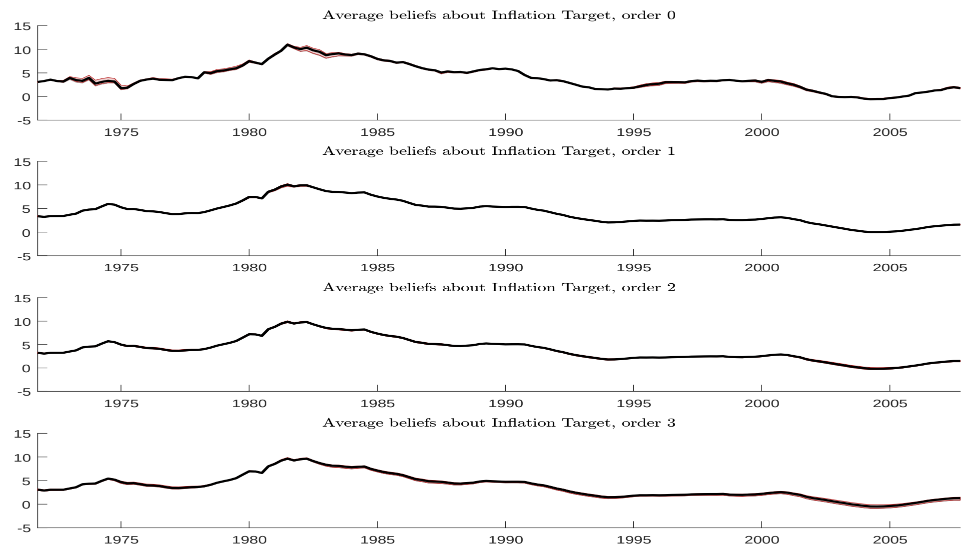
Sources of higher-order wedge in 3-year bond yields
Annualized Percent
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1980
1990
2000
g_r
g_π
g_y
τ
υ



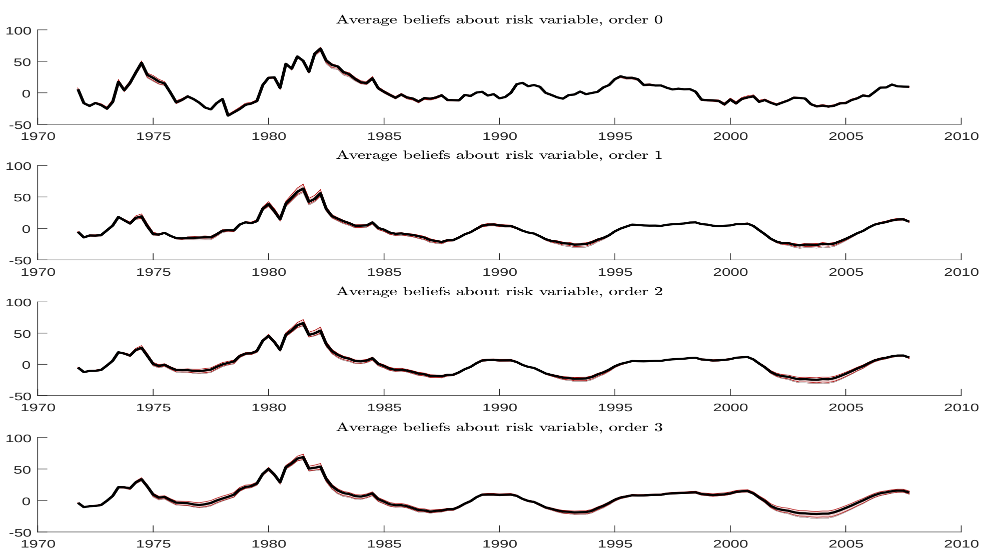
Sources of higher-order wedge in 4-year bond yields
Annualized Percent
1.5
1
0.5
0
-0.5
-1
-1.5
1980
1990
2000
g_r
g_π
g_y
τ
υ


Sources of higher-order wedge in 5-year bond yields
Annualized Percent
1.5
1
0.5
0
-0.5
-1
1980
1990
2000
g_r
g_π
g_y
τ
υ

D.2 State Estimates and Yield Decompositions



(a) Filtered estimate of inflation target and first three orders of expectation an annualized percent, dispersed information model.



(b) Filtered estimate of risk variable and first three orders of expectation, dispersed information model.

Internet appendix – not intended for publication

D.3 Results, full information model

Table 10: Posterior Estimates, Full Information Model

	Mode	Mean	Median	5th percentile	95th percentile	Std. Dev
ϕ_r	0.5349	0.5373	0.5350	0.5154	0.5727	0.0170
ϕ_π	0.1771	0.1760	0.1702	0.1098	0.2522	0.0435
ϕ_y	0.1178	0.1114	0.1106	0.0908	0.1353	0.0133
ϕ_v	0.0283	0.0221	0.0221	0.0128	0.0314	0.0058
σ_r	0.0020	0.0021	0.0021	0.0019	0.0023	0.0001
ρ_{yr}	-0.9946	-0.9278	-0.9334	-1.1513	-0.6789	0.1454
$\rho_{y\pi}$	-0.3899	-0.4031	-0.4077	-0.5526	-0.2503	0.0943
ρ_{yy}	0.9525	0.9277	0.9266	0.8835	0.9730	0.0272
ρ_{yv}	-0.0013	-0.0149	-0.0131	-0.0368	-0.0011	0.0109
$\sigma_{y\pi}$	0.2903	0.3863	0.3868	0.1419	0.6309	0.1509
$\sigma_{y\tau}$	2.6851	2.6387	2.6360	2.2813	2.9877	0.2174
σ_y	0.0066	0.0068	0.0068	0.0061	0.0075	0.0004
σ_τ	0.0012	0.0012	0.0012	0.0010	0.0013	0.0001
$\rho_{\pi r}$	0.8326	0.8344	0.8280	0.7050	1.0022	0.0831
$\rho_{\pi\pi}$	0.4024	0.4059	0.4057	0.3579	0.4558	0.0299
$\rho_{\pi y}$	-0.1750	-0.1787	-0.1774	-0.2191	-0.1440	0.0235
$\rho_{\pi v}$	-0.0842	-0.0832	-0.0822	-0.1005	-0.0706	0.0088
$\sigma_{\pi\tau}$	-0.1585	-0.1803	-0.1836	-0.3356	-0.0191	0.0956
σ_π	0.0040	0.0040	0.0040	0.0036	0.0044	0.0003
ρ_{vv}	0.8610	0.8550	0.8533	0.8321	0.8931	0.0177
σ_{vr}	9.8381	9.7069	9.7540	9.2463	9.9673	0.2172
$\sigma_{v\pi}$	2.1456	1.9804	2.0660	1.2514	2.5665	0.4220
σ_{vy}	-2.1201	-2.1669	-2.1473	-2.6129	-1.7968	0.2609
$\sigma_{v\tau}$	0.8711	0.8384	0.7641	0.1380	1.6089	0.4336
λ_r	1.2591	1.1900	1.2892	-0.3004	2.2988	0.7652
λ_π	-4.3221	-4.4279	-4.7589	-7.6234	0.1405	2.3402
λ_y	-0.6214	-0.2257	-0.4712	-2.4933	2.3231	1.5115
λ_τ	-0.1198	-0.1123	-0.1139	-0.2503	0.0229	0.0960
λ_r^x	18.3449	18.5892	18.6880	13.1336	23.8793	3.3348
λ_π^x	-76.2955	-77.1364	-76.8624	-80.6353	-73.8037	2.0341
λ_y^x	-17.4764	-22.7287	-21.9104	-33.4025	-17.1115	4.4454
λ_τ^x	0.1536	0.0564	0.0529	-1.7520	1.8245	1.0744
$\widetilde{\sigma}_4$	0.0012	0.0012	0.0012	0.0011	0.0014	0.0001
$\widetilde{\sigma}_8$	0.0011	0.0011	0.0011	0.0010	0.0012	0.0001
$\widetilde{\sigma}_{12}$	0.0009	0.0010	0.0010	0.0008	0.0011	0.0001
$\widetilde{\sigma}_{16}$	0.0010	0.0009	0.0009	0.0008	0.0011	0.0001
$\widetilde{\sigma}_{20}$	0.0010	0.0010	0.0010	0.0008	0.0011	0.0001
$\widetilde{\sigma}_{40}$	0.0014	0.0014	0.0014	0.0012	0.0016	0.0001
$\widetilde{\sigma}_\pi^1$	0.0032	0.0032	0.0032	0.0031	0.0032	0.0000
$\widetilde{\sigma}_\pi^4$	0.0039	0.0039	0.0039	0.0038	0.0039	0.0000

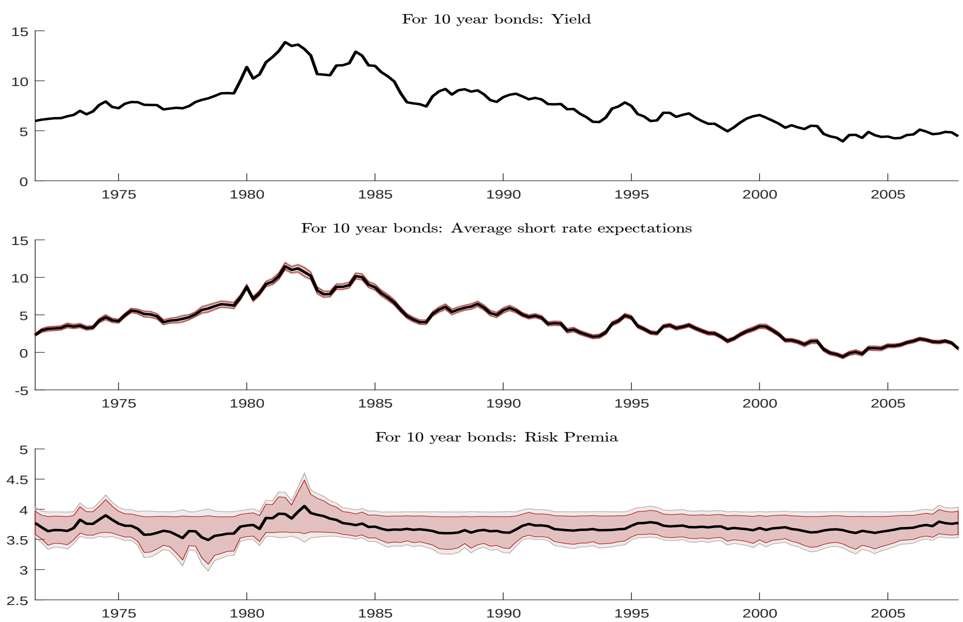
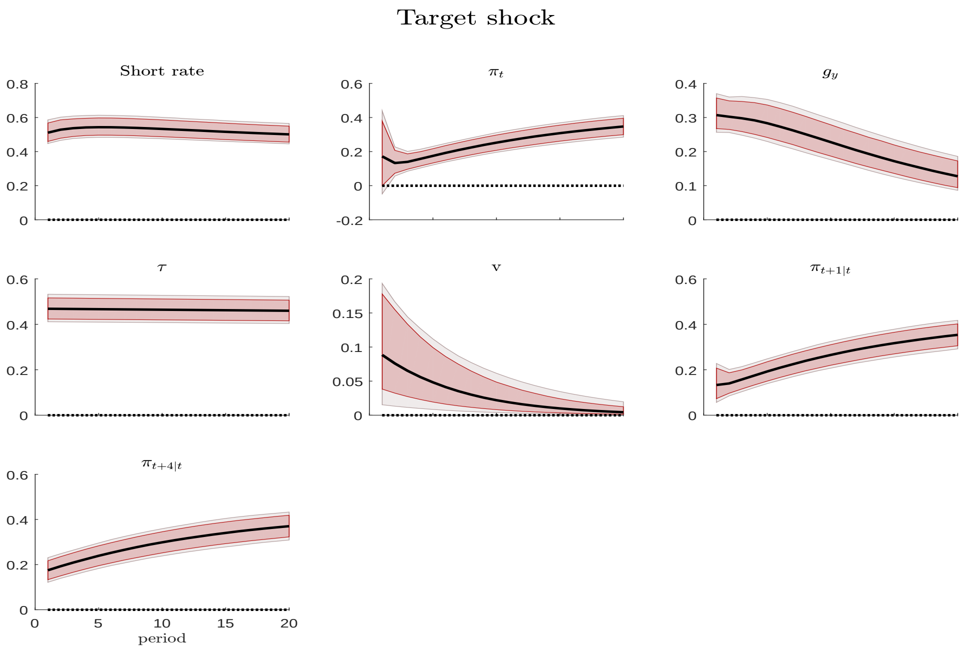
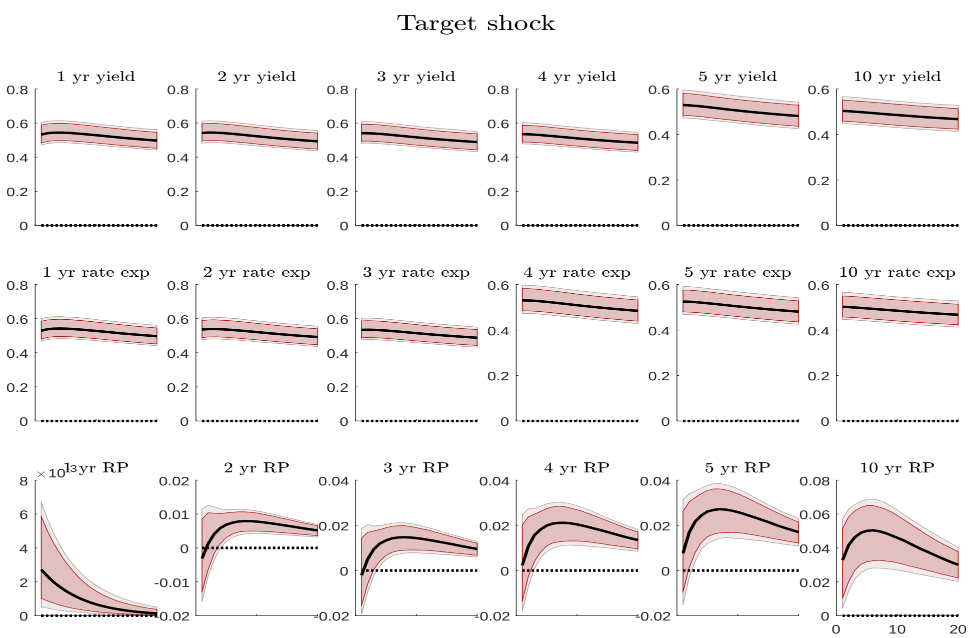


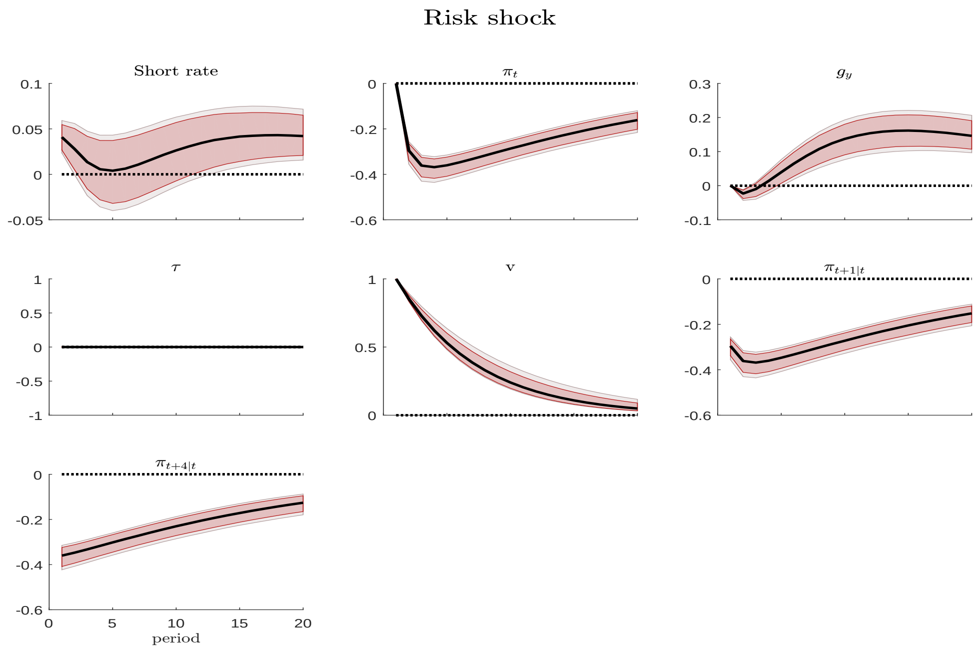
Figure 16: Decomposition of 10 year yields, full information



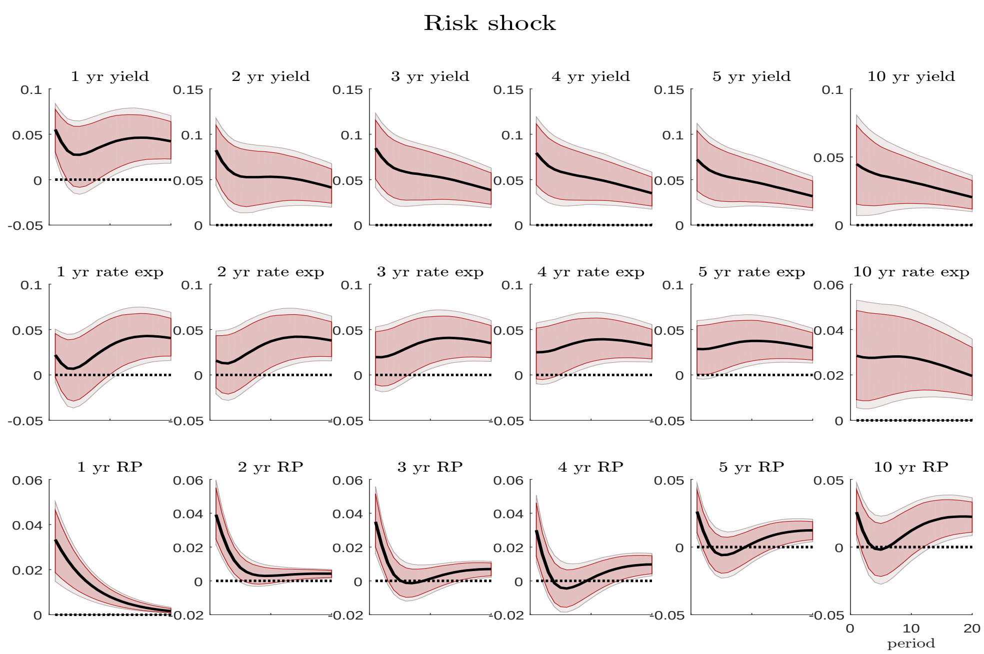
(a) Response of non-financial variables to inflation target shock, full information



(b) Response of financial variables to inflation target shock, full information



(a) Response of non-financial variables to risk shock, full information



(b) Response of financial variables to risk shock, full information